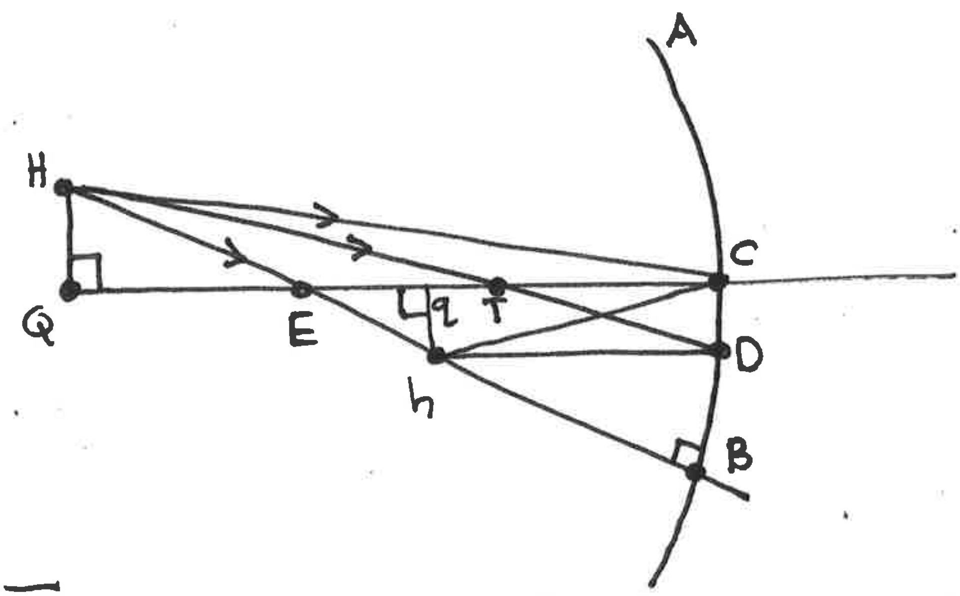
ASG vol 3 Ex 12.7 (Spherical mirrors and the thin-lens equation)



a) $\overline{HB}$ passes thru E & reflects straight back since it hits the mirror at a 90° angle (Axiom II)

b) $\measuredangle HCQ = \measuredangle hCQ$ also by Axiom II, since QC is $\perp$ to the mirror's surface

c) Since T is the focal point of the spherical mirror (i.e. $\overline{TC} = \overline{EC}/2$, by Ex. 12.6), $\overline{hD}$ must be parallel to $\overline{QC}$. This follows from the fact that an incoming ray along $\overline{hD}$ reflects to point T (see Ex 12.6) and by Axiom II.

e) • Is it true that $\frac{QC}{QH} \overset{?}{=} \frac{qC}{qh}$

• Since $\measuredangle HCQ = \measuredangle qCH$, $\Delta HCQ \sim \Delta qCH$

• Thus the ratio of the long side to short side is equal

• Therefore $\boxed{\frac{QC}{QH} = \frac{qC}{qh}}$ ✓

• But is $\frac{TC}{qh} \overset{?}{=} \frac{TQ}{QH}$

• First, drop a perpendicular from T down to $\overline{hD}$. Call it x.

• Then $\Delta TDx \sim \Delta HTQ$. Again, the ratio of sides are equal, so (since $\overline{xD} \approx \overline{TC}$ for rays near the axis)

$$\boxed{\frac{TC}{qh} = \frac{TQ}{QH}}$$ ✓

• Combining these gives

$$\frac{QC}{qC} = \frac{QH}{qh} = \frac{TQ}{TC} \iff \boxed{\frac{QC}{qC} = \frac{TQ}{TC}}$$ ✓

• Now, since $TC = TE$ and since $\left\{ \begin{array}{l} QC = TQ + TC \\ \text{and } qC = qT + TC \end{array} \right\}$, we have

$$\frac{TQ + TE}{qT + TE} = \frac{TQ}{TE}$$

- Inverting both sides gives

$$\frac{qT + TE}{TQ + TE} = \frac{TE}{TQ}$$

$$\frac{TE\left(\frac{qT}{TE} + 1\right)}{TQ + TE} = \frac{TE}{TQ}$$ ← factoring out TE from previous equation

$$\cancel{TE}\left(\frac{qT}{TE} + 1\right) = \frac{\cancel{TE}}{TQ}\left(TQ + TE\right)$$

↓ simplifying

$$\cancel{1+}\frac{qT}{TE} = \cancel{1+}\frac{TE}{TQ}$$

$$\boxed{\frac{qT}{TE} = \frac{TE}{TQ}}$$ ✓ ← proven!

f) $\frac{Tq}{TE} = \frac{TE}{TQ} \Rightarrow (Tq)(TQ) = TE^2$

define $f = TE = TC$

$d_o = QC$

$d_i = qC$

Now $Tq = qC - TC = d_i - f$

And $TQ = QC - TC = d_o - f$

Thus $\boxed{(d_i - f)(d_o - f) = f^2}$

g) $(d_o - f)(d_i - f) = f^2$

$$d_o d_i + \cancel{f^2} - f d_o - f d_i = \cancel{f^2}$$

$$\frac{d_o d_i}{f d_o d_i} = \frac{f d_o}{f d_o d_i} + \frac{f d_i}{f d_o d_i}$$

$$\boxed{\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}}$$ ✓

h) The magnification is $M = \frac{qh}{QH}$

By similar triangles $\Delta HQC \sim \Delta hqC$

we know $\frac{HQ}{QC} = \frac{hq}{qC}$

or $\frac{qC}{QC} = \frac{hq}{QH}$

or $\frac{qh}{QH} = \frac{d_i}{d_o}$

or $\boxed{M = \frac{d_i}{d_o}}$