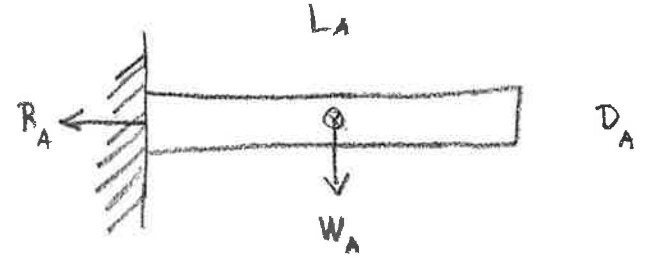
Ex 7.1) Limestone pillar problem



This limestone piller is about to break

$D_A = 1$ ft

$L_A = 20$ft

a.) How thick must one make a 30 foot piller so it is about to break under its own weight? Well, comparing two prisms of different lengths and thicknesses

$$\frac{D_B}{D_A} = \frac{W_B L_B / R_B}{W_A L_A / R_A}$$ (using law of lever for each prism)

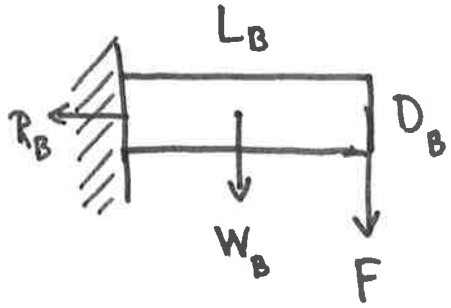
$$= \left(\frac{g\rho_B D_B^2 L_B}{g\rho_A D_A^2 L_A}\right)\left(\frac{L_B}{L_A}\right)\left(\frac{R_A}{R_B}\right)$$ (using formula for weight of prisms)

$$= \left(\frac{D_B^2 L_B^2}{D_A^2 L_A^2}\right)\left(\frac{D_A^2}{D_B^2}\right)$$ (since $R \propto D^2$)

$$\frac{D_B}{D_A} = \left(\frac{L_B}{L_A}\right)^2 = \left(\frac{30}{20}\right)^2 = \frac{9}{4}$$

So the diameter of B is **2 ¼ feet**

Ex 7.1 b)



Force F applied to end of limestone pillar so that it's about to break. What is F?

For equilibrium: $R_B \frac{D_B}{2} = W_B \frac{L_B}{2} + F L_B$

(support moment) = (breaking moments)

Solve for $F = \frac{R_B D_B}{2L_B} - \frac{W_B L_B}{2 L_B} = \frac{R_B D_B}{2L_B} - \frac{W_B}{2}$

but I know that for a different prism, A, which is about to break under its own weight

$$R_A \frac{D_A}{2} = W_A \frac{L_A}{2}$$

and also that $D_A = D_B$ (prisms have same diameter)

and that $R_A = R_B$ (same material & diameter)

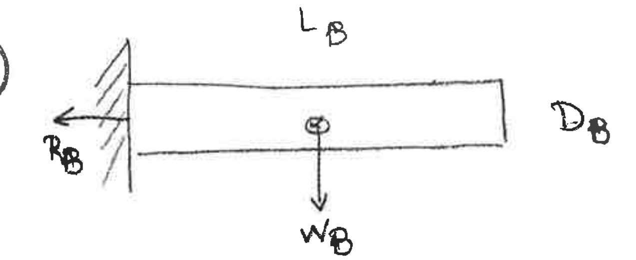
So $F = \frac{W_A L_A}{2L_B} - \frac{W_B}{2}$. But $L_A = 2L_B$

also, $W_A = \frac{150 \text{ lbs}}{\text{cu.ft}} \times 20 \text{ cu.ft} = 3000 \text{ lbs}$

$W_B = 150 \text{ lbs/cu.ft} \times 10 \text{ cu.ft} = 1500 \text{ lbs}$

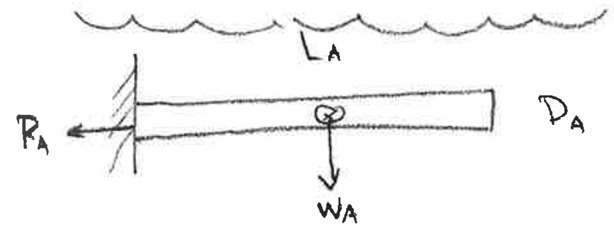
So $F = W_A - W_B/2 = \boxed{2250 \text{ lbs}}$

7.1 E)



This beam is about to break under its own weight.

$D_B = 1$ foot, $L_B = 20$ feet



This is a submerged beam about to break under its own weight. $L_A = 20$ feet

$D_A = ?$

$$\frac{D_A}{D_B} = \frac{W_A L_A R_B}{W_B L_B R_A}$$ (same as in problem 1.)

$$= \frac{(gV_A \rho_l - gV_A \rho_w)}{gV_B \rho_l} \frac{L_A}{L_B} \frac{R_B}{R_A}$$

(since the weight of the submerged beam is reduced by buoyancy.

ρ_l = density of limestone

ρ_w = density of water

$$= \frac{D_A^2 L_A (\rho_l - \rho_w)}{D_B^2 L_B \rho_l} \frac{L_A}{L_B} \frac{D_B^3}{D_A^3}$$ simplifying

$$\frac{D_A}{D_B} = \left(\frac{L_A}{L_B}\right)^2 \left(1 - \frac{\rho_w}{\rho_l}\right)$$

$$= (1)^2 \left(1 - \frac{62\ \text{lbs/ft}^3}{150\ \text{lbs/ft}^3}\right) = \boxed{0.59\ \text{feet}}$$