

ASG v2 Ex 2.3 (Polygon paradox)

Referring to Fig 2.2, we find that

a) $\angle AGB = 60$ degrees (since ABCDEF is a hexagon)

If $\overline{AG} = 1$ and $\overline{AG} = 2\overline{AI}$ then

$$\overline{HI} = \frac{1}{2}, \quad \overline{IO} = \frac{1}{2}, \quad \overline{AS} = 6$$

b) There are 5 gaps in segment \overline{HT} .

Excluding these gaps, $\overline{HT} = 6\overline{HI} = 3$ and

$\frac{\overline{HT}}{\overline{AS}} = \frac{1}{2}$. If we include the gaps, then

$$\frac{\overline{HT}}{\overline{AS}} = \frac{11}{12}.$$

c) There are 5 gaps in \overline{GV} . Excluding the gaps, $\overline{GV} = 6$

Including the gaps, $\overline{GV} = 5$.

d) For a 12-sided polygon, there would be 11 gaps in \overline{HT} .

\overline{HT} would be $12\overline{HI}$, and $\overline{HI} = \sin(15^\circ)$. Since $\overline{AB} = 2\sin 15^\circ$, we would still have

$$\frac{\overline{HT}}{\overline{AS}} = \frac{1}{2} \text{ (excluding gaps)} \quad \text{or} \quad \frac{23}{24} \text{ (including gaps)}$$

e) For a circle, there would be an infinite number of gaps.

So the inner circle, when rolling, travels a farther distance than its circumference.

f) This implies that an apparently continuous substance can have "holes" in it. Perhaps solid matter has holes, too?