Chapter 7: Eclipses

We can now apply our Sun shadow knowledge from the previous chapter to the Earth and Moon so we can understand these shadows during eclipses.

We have seen that eclipses can only occur when the Moon crosses the ecliptic at its nodes about every six months. Using what we know about shadows and the sizes and distances of the Earth, Sun and Moon, we can calculate how close to a node the Sun and Moon must be to generate eclipses of various kinds.

During a lunar eclipse the Moon passes through Earth's shadow, and during a solar eclipse the Moon's shadow falls upon Earth.

We can calculate the tapers of the umbras and penumbras for each case. We will use modern values for sizes and distances (Figure 7.1), rather than the estimates of the ancient Greek astronomers.*

Notice that the Sun is so much larger than both the Earth and Moon (and the soccer ball) that the difference in the angle of the shadows is too small for us to measure.

First we'll look at the umbra of Earth with the Moon passing through, as shown in Figure 7.2 on the following page. Starting with the difference in radius between the Sun and Earth:

$$430{,}000 - 4{,}000 = 426{,}000 \text{ miles}$$

$$\sin\theta = 426{,}000 \div 93{,}000{,}000$$

$$\theta \approx \tfrac{1}{4}^\circ$$

Next we determine the penumbra of Earth:

$$\sin(\alpha / 2) = 430{,}000 \div 93{,}000{,}000$$

$$\alpha \approx \tfrac{1}{2}^\circ$$

Now we'll determine the umbra of the Moon, which falls on Earth, as shown in Figure 7.3. Starting with the difference in radius between the Sun and the Moon:

$$430{,}000 - 2{,}000 = 428{,}000 \text{ miles}$$

$$\sin\theta = 428{,}000 \div 93{,}000{,}000$$

$$\theta \approx \tfrac{1}{4}^\circ$$

And for the Moon's penumbra:

$$\sin(\alpha / 2) = 430{,}000 \div 93{,}000{,}000$$

$$\alpha \approx \tfrac{1}{2}^\circ$$

	modern (miles)	Rounded (miles)
radius of Earth	3960	4,000
radius of Moon	1080	1,000
radius of Sun	432,687	430,000
distance Earth-Moon	238,607	240,000
distance Earth-Sun	92,957,130	93,000,000

Figure 7.1: Modern average distances and the rounded values we will use to illustrate principles.

* *The purpose of this chapter is to explain the phenomena of eclipses, including the idea of eclipse seasons and the celestial mechanics that cause the seasons to occur. Approximations and rounded values will be used to simplify the calculations. These approximations and rounded values will affect the final results, i.e. the lengths of each season, but the results are still quite accurate. Most of the season lengths are within 5%, with the farthest outlier being around 15% different from the actual historical data.*

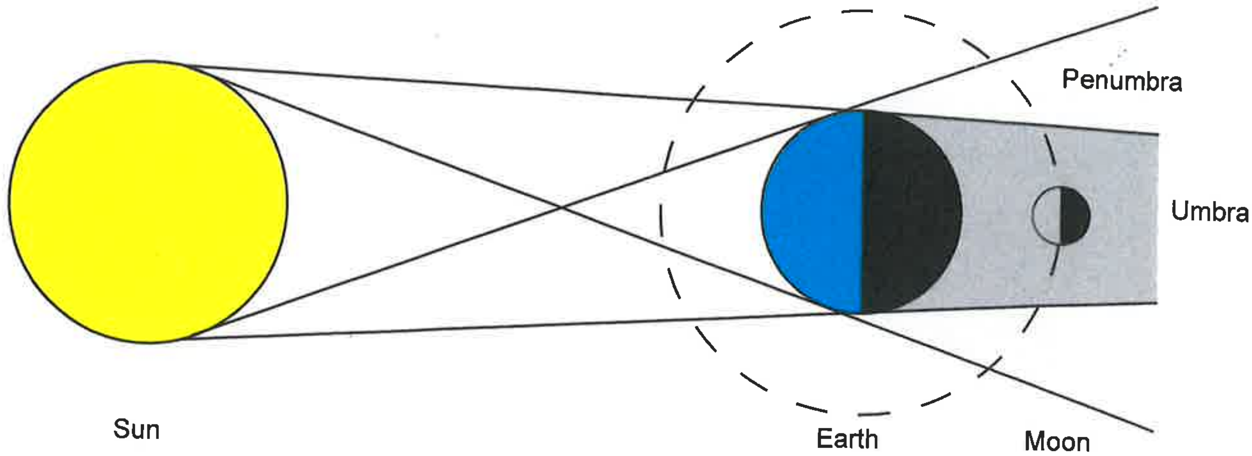


Figure 7.2: During a lunar eclipse, the Moon travels through Earth's shadow

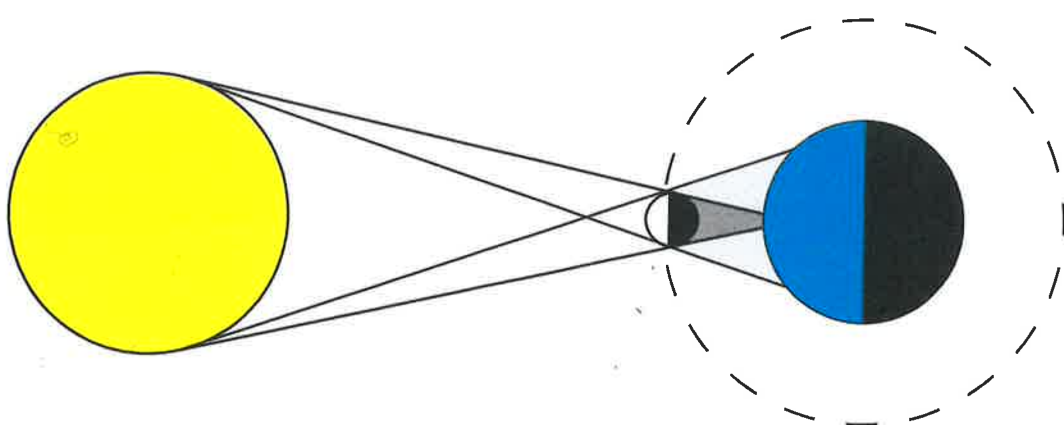


Figure 7.3: During a solar eclipse, the Moon's shadow falls on Earth

Since we know the Earth's umbra tapers in by ¼° per side and we know the distance to the Moon, as shown below, we can calculate the difference in radius between the umbra where the Moon crosses and the Earth itself:

$$\sin 1/4° = \Delta r \div 240{,}000 \text{ miles}$$

$$\Delta r \approx 1000 \text{ miles}$$

Since Earth has a diameter of 8000 miles, and the shadow is 2 × 1000 miles smaller, the diameter of the umbra where the Moon crosses is about 6000 miles, which is 3 times the diameter of the Moon.

The Earth's penumbra tapers out from the edge of the umbra by ½°, which is ¼° out from straight, as shown in Figure 7.5 on the following page. The penumbra tapers out the same amount as the umbra tapers in from the actual size of the Earth, 1000 miles per side.

Adding 2 × 1000 to the Earth's diameter gives a penumbral diameter of 10,000 miles where the Moon crosses, 5 times the diameter of the Moon. This is summarized and illustrated in Figure 7.6.

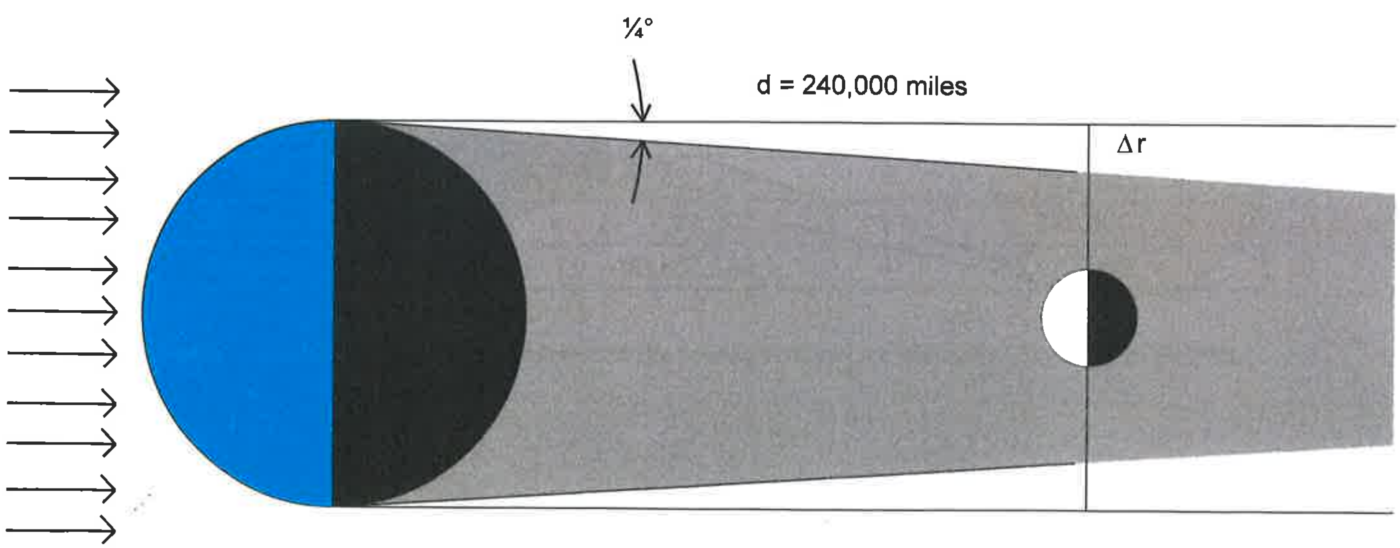


Figure 7.4: The Earth's umbra tapers down by ¼° per side

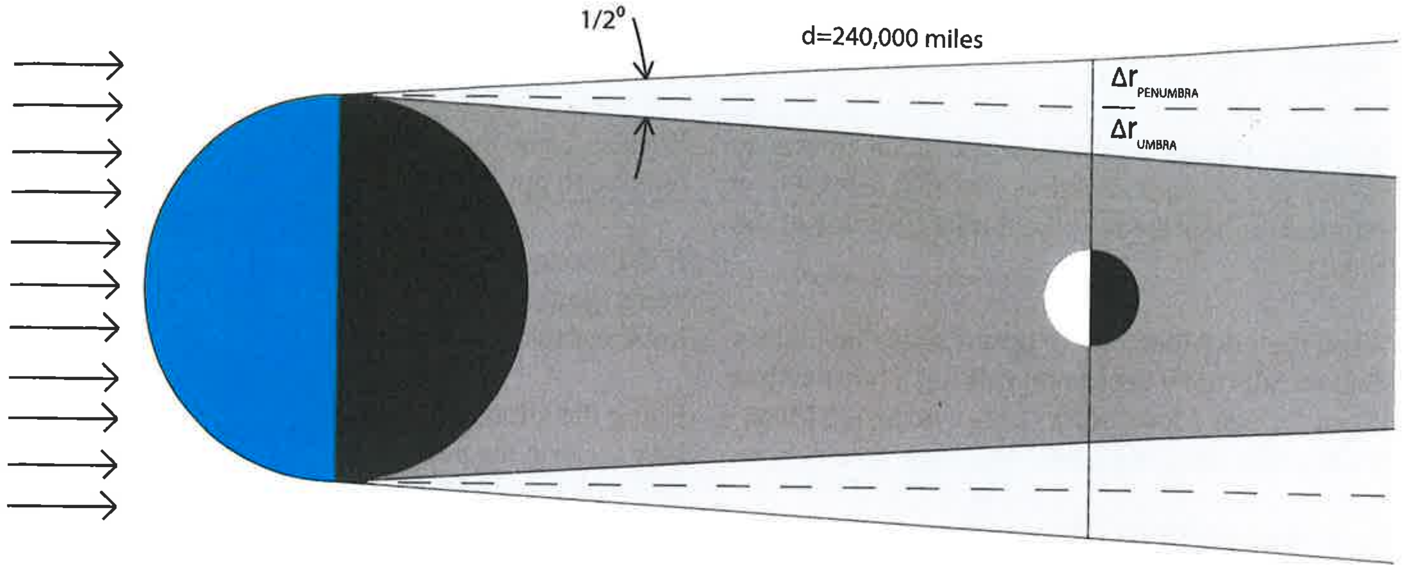


Figure 7.5: The Earth's penumbra tapers out ½° from the edge of the umbra

	Diameter in Miles	Moon Diameters	Angular Size
Moon	2,000	1	½°
Earth's Umbra at Moon distance	6,000	3	1½°
Earth's Penumbra at Moon distance	10,000	5	2½°

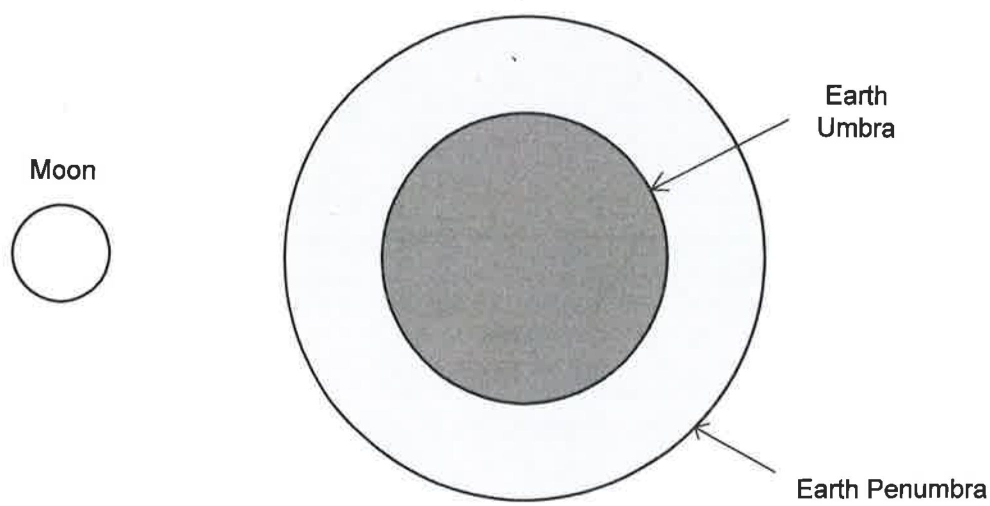


Figure 7.6: Comparative sizes of Moon, Earth's Umbra, and Earth's Penumbra

In earlier chapters we saw how the Moon crosses the ecliptic at a 5° angle. Eclipses can only happen when the Moon is near the ecliptic, that is, near one of the nodes.

When the full Moon occurs near a node, the Earth's shadow falls upon the Moon, causing a lunar eclipse. When the new Moon occurs near a node, the Moon's shadow falls upon the Earth, causing a solar eclipse.

How close to a lunar node do the Sun and Moon have to be in order for an eclipse to occur?

Figure 7.7 shows the Moon and the Earth's umbra approaching the descending node.

We know that the Moon has an angular size of ½°, and the umbra of Earth has an angular diameter of 1½°.

We also know that the Moon's path is inclined 5° relative to the ecliptic.

If the Moon passes Earth's umbra near enough to the node, the shadow falls on the Moon and we have a lunar eclipse. See Figure 7.8 on the following page.

Using the information we have, we can calculate how close to the node the Moon and shadow must be to overlap (see Figure 7.9):

$$\sin 5° \approx x / d$$

$$d = x / \sin 5°$$

$$d = (\tfrac{1}{4}° + \tfrac{3}{4}°) \div 0.083$$

$$d = 12°$$

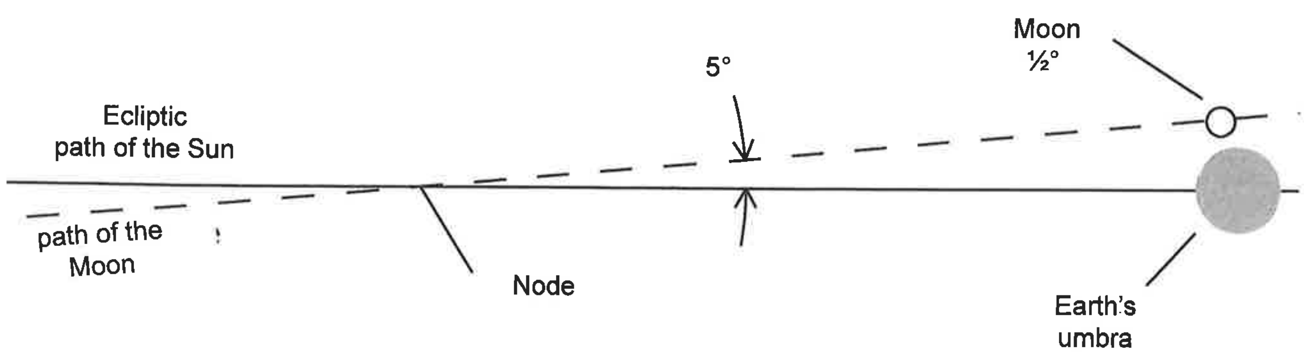


Figure 7.7: The Moon and Earth's umbra approach the descending node

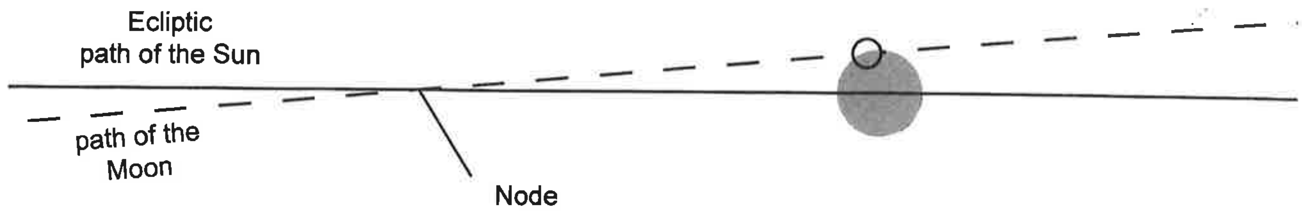
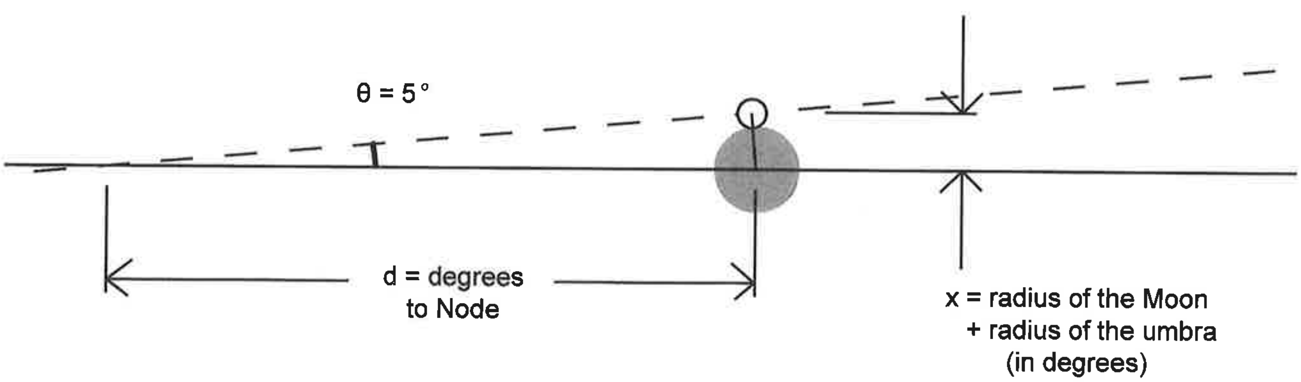


Figure 7.8: The Earth's shadow causing a lunar eclipse



$$\sin 5^\circ \approx x / d$$

$$d = x / \sin 5^\circ$$

$$d = (\tfrac{1}{4}^\circ + \tfrac{3}{4}^\circ) \div 0.083 = 12^\circ$$

Figure 7.9: Calculating eclipse zone

When Earth's umbra is within 12° of a lunar node when the Moon passes by, the umbra overlaps the Moon causing a lunar eclipse.

Since the Sun, and therefore the Earth's shadow, moves about 1° per day along the ecliptic, the shadow is in eclipse position for about 24 days, 12 days before it reaches the node, and 12 days after, as shown below.

Since there are about 30 days between full Moons, a full Moon may or may not occur during these 24 days. The 24-day period when Earth's shadow is near the lunar node is called a *lunar eclipse season.* During each season, there may be either zero or one lunar eclipse, but there can never be two lunar eclipses in one season.

If a full Moon occurs while the Sun, and therefore the Earth's shadow, is within the 24-day window near the node, then an eclipse occurs. Refer to full Moon #2 in Figure 7.11 on the following page.

The previous full Moon occurred 30 days prior, so it must be out of the eclipse zone, shown by full Moon #1 in the same figure.

The next full Moon also must be out of the zone, as shown by full Moon #3.

In this case, there is one partial or total lunar eclipse during the season, depending on if the Moon passes completely within the shadow or just grazes it.

If a full Moon occurs within 6 days before the eclipse zone, then the next full Moon 30 days later will also be out of the eclipse zone. In this case there is no eclipse during the season. See Figure 7.12.

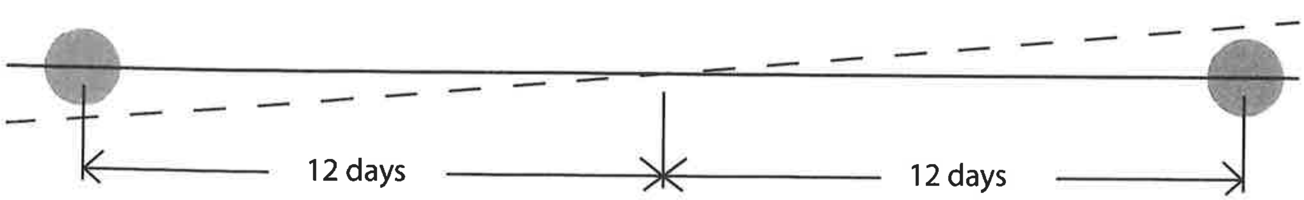


Figure 7.10: A lunar eclipse can happen 12 days before or after Earth's shadow reaches the node

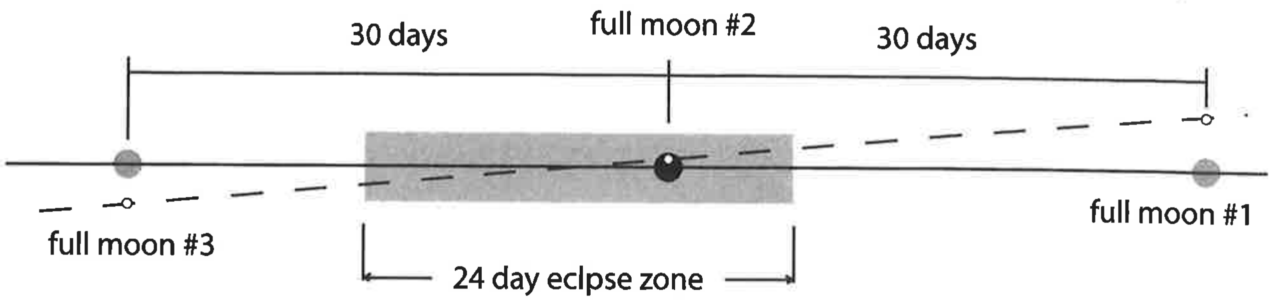


Figure 7.11: The full Moon occurring in the eclipse zone

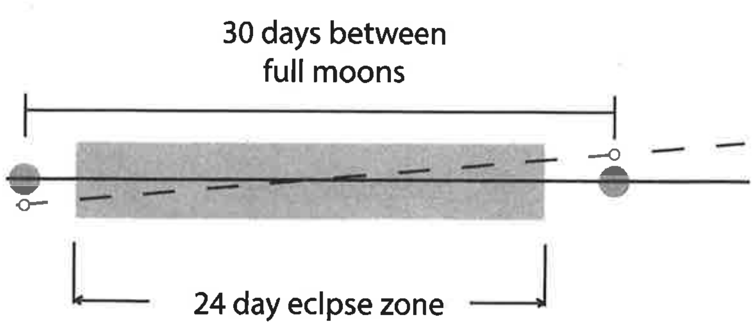


Figure 7.12: If full Moon occurs at the wrong time, no eclipse occurs

We have seen that a lunar eclipse can only occur when a full Moon is within 12 days (or degrees) of the lunar node. What about a solar eclipse?

A solar eclipse occurs when the Moon's shadow falls on Earth. Solar eclipses can only happen during new Moons when the Sun is near a lunar node.

In order to find the solar eclipse zone, we need to know the size of the Moon's shadow when it reaches Earth.

The Moon's shadow tapers practically the same amount as Earth's. Refer to Figure 7.13 below. The umbra tapers in by ¼°, and the penumbra tapers out from the umbra by ½°, therefore:

$$\sin \tfrac{1}{4}° = \Delta r \; / \; 240{,}000 \text{ miles}$$

$$\Delta r \approx 1000 \text{ miles}$$

Since the Moon has a diameter of 2000 miles, and its umbra tapers by 2 × 1000 miles, the umbra barely reaches Earth.

The penumbra tapers out about the same amount, so it is:

$$2000 + 2 \times 1000 = 4000 \text{ miles}$$

About 4000 miles in diameter, or about half the diameter of Earth. See Figure 7.14 on the following page.

The Moon's umbra tapers down to almost a single point when it reaches Earth. In fact, during some eclipses the umbra never reaches Earth. These are called *annular eclipse* because you can see the outside ring, or annulus, of the Sun during the eclipse. See Figure 7.15.

During most solar eclipses, the Moon's umbra is a a couple of hundred miles in diameter.

The penumbra is much larger, about half of Earth's diameter. See the summary in Figure 7.16 on the following pages.

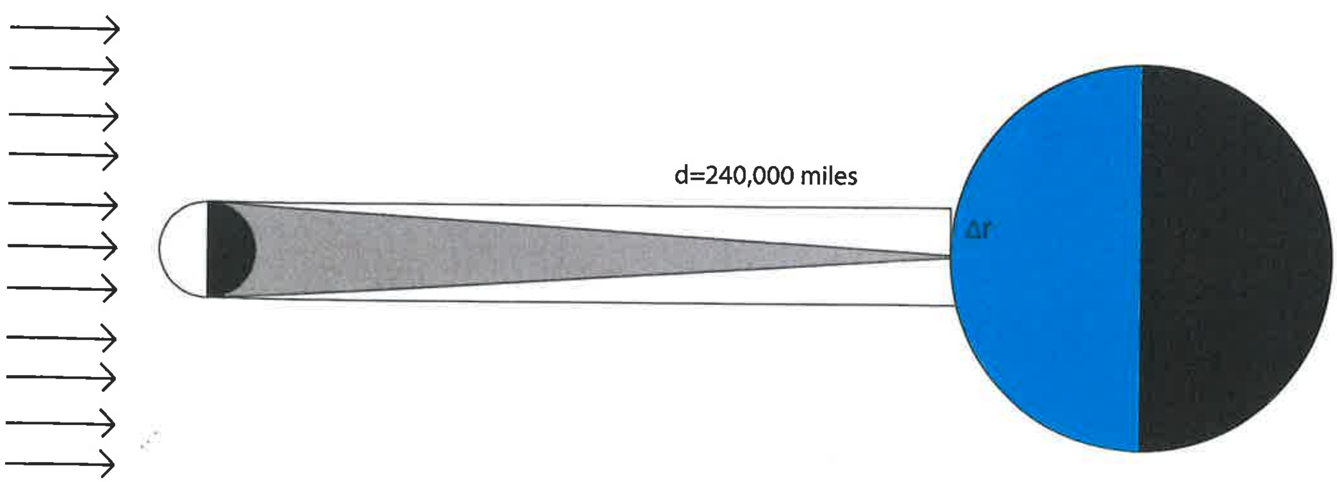


Figure 7.13: The Moon's umbra tapers down by ¼° per side

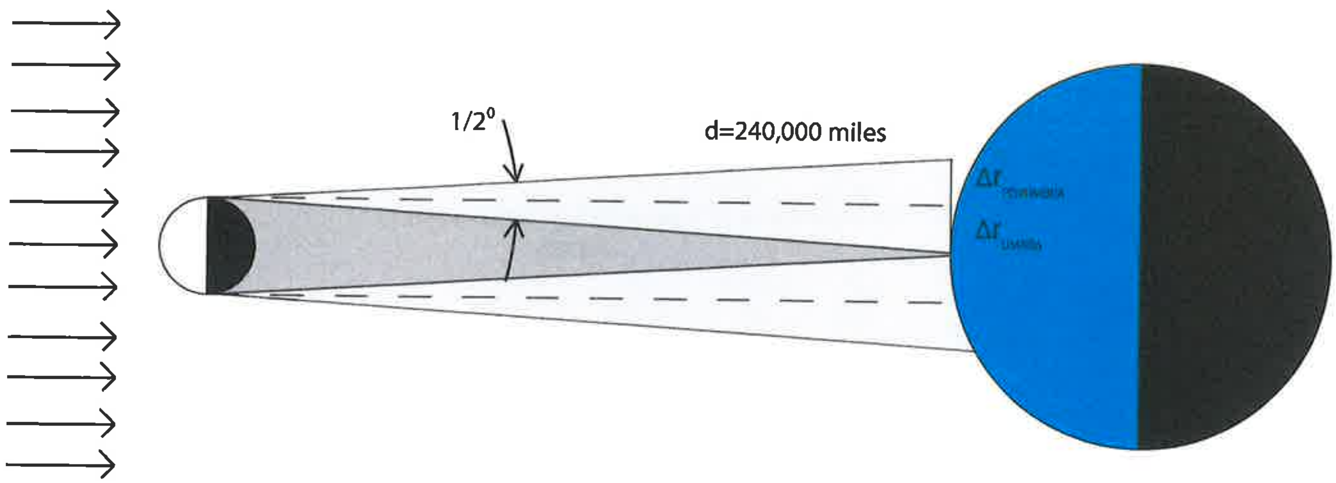


Figure 7.14: The Moon's penumbra tapers out ½° from the edge of the umbra

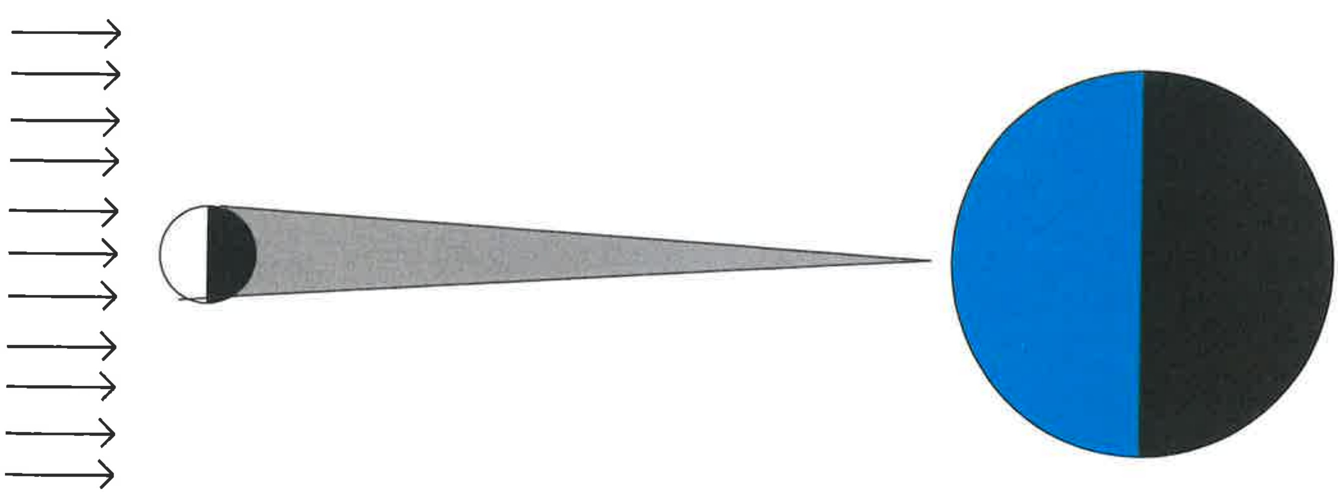


Figure 7.15: During an annular eclipse, the Moon's umbra doesn't quite reach the Earth

	Diameter (miles)	Moon-diameters
Earth	8,000	4
Moon's Umbra at Earth	0	0
Moon's Penumbra at Earth	4,000	2

Figure 7.16: Summary of umbra and penumbra sizes

We can calculate the length of the solar eclipse season by assuming that during an average eclipse the Moon's umbra is a point as it reaches Earth. See Figure 7.17 on the following page.

When the Moon's umbra just touches Earth, the Moon must be one Earth radius, or two moon-diameters, above or below the ecliptic

Since the Moon appears to be ½°, two moon-diameters is 1°. A solar eclipse occurs if the new Moon is within 1° of the ecliptic.

The length of the solar eclipse season is the same as the length of the lunar eclipse season, as shown in Figure 7.18. The solar eclipse season is 24 days, 12 days before the node and 12 days after.

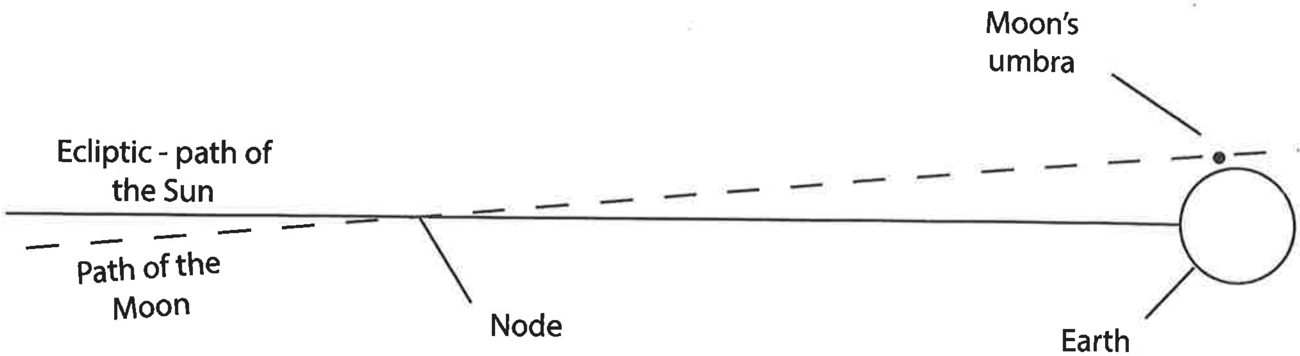
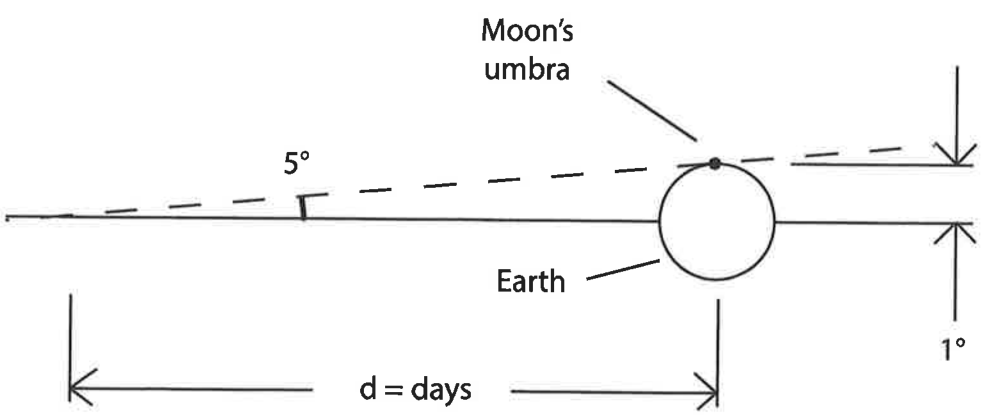


Figure 7.17: Finding the length of the solar eclipse season



$$\sin 5° \approx 1° / d$$

$$d \approx 1° / \sin 5°$$

$$d \approx 12°$$

Figure 7.18: The length of the solar eclipse season is same as length of lunar eclipse season

Types of Eclipses

So far we have found that there is a 24-day period around a lunar node for both lunar and solar eclipses. We assumed that a lunar eclipse occurs when the Earth's umbra touches the Moon, and that a solar eclipse occurs when the Moon's umbra touches Earth.

But there are different types of lunar and solar eclipses, and they are defined in different ways. A total lunar eclipse occurs when the Moon is completely within Earth's umbra, as shown in the first illustration in Figure 7.19. A total solar eclipse occurs when the Moon's umbra falls on Earth. Refer to the first illustration in Figure 7.20.

A partial solar eclipse is a different phenomena than a partial lunar eclipse. Refer to the second illustration in the same figures. During a partial lunar eclipse, part of the Moon touches Earth's umbra. A partial solar eclipse occurs when the Moon's penumbra touches Earth.

Finally, a penumbral lunar eclipse occurs when part of the Moon touches Earth's penumbra, as shown in the third illustration in Figure 7.19. Note that you can't readily observe a penumbral lunar eclipse because the dimming is faint.

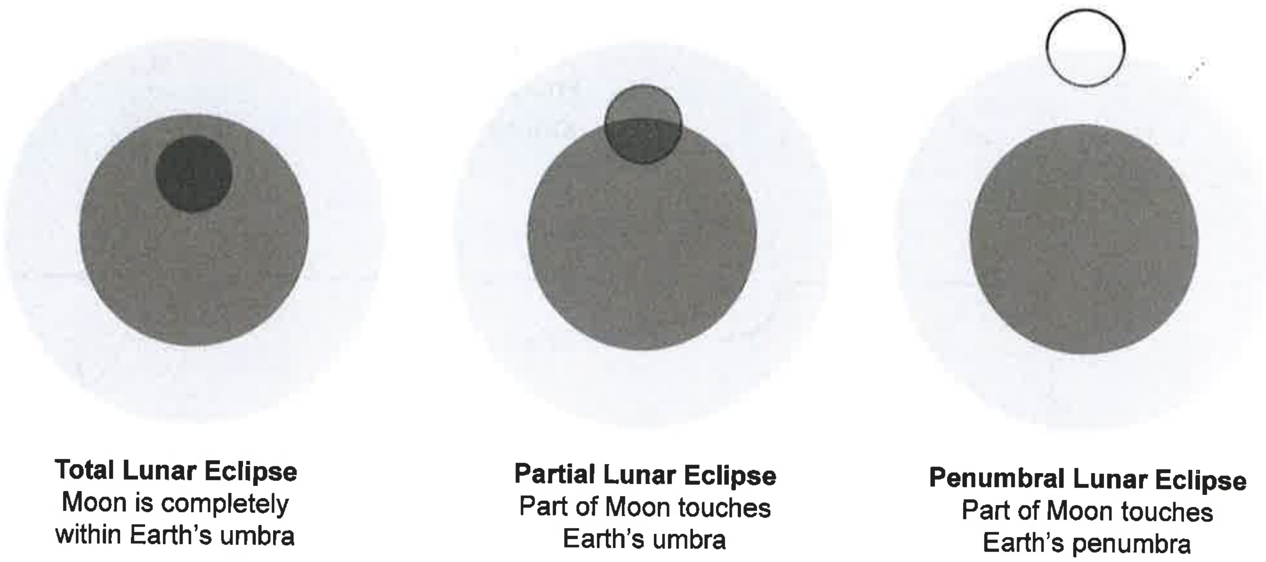


Figure 7.19: Types of lunar eclipses

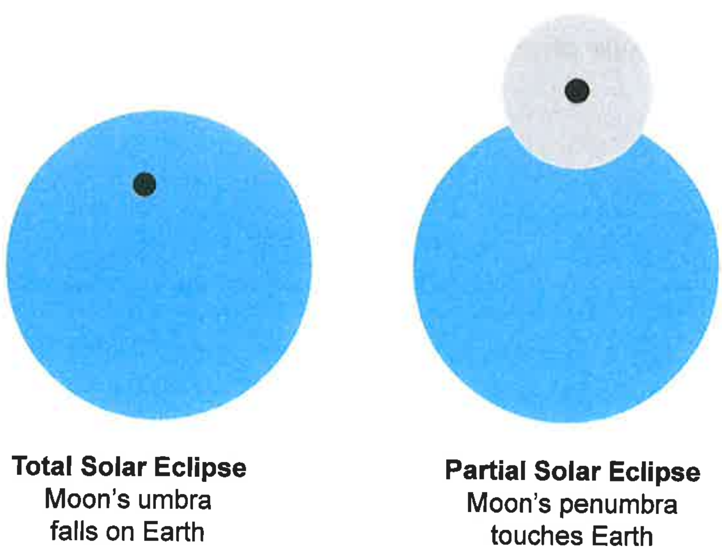


Figure 7.20: Types of solar eclipses

We found that the length of the total solar eclipse season is the same as the length of the partial and total lunar eclipse season, 24 days (12 days before and 12 days after the node).

A partial solar eclipse can happen farther from the node, because the Moon's penumbra is a full degree rather than the point of the umbra. This means the partial eclipse season extends out another 6 days farther from the node than total eclipse season, as shown on the following page in Figures 7.21 and 7.22.

The partial solar eclipse season is 12 days total, 6 on each side of the total eclipse season. The total season for solar eclipses is 36 days, 24 total and 12 partial.

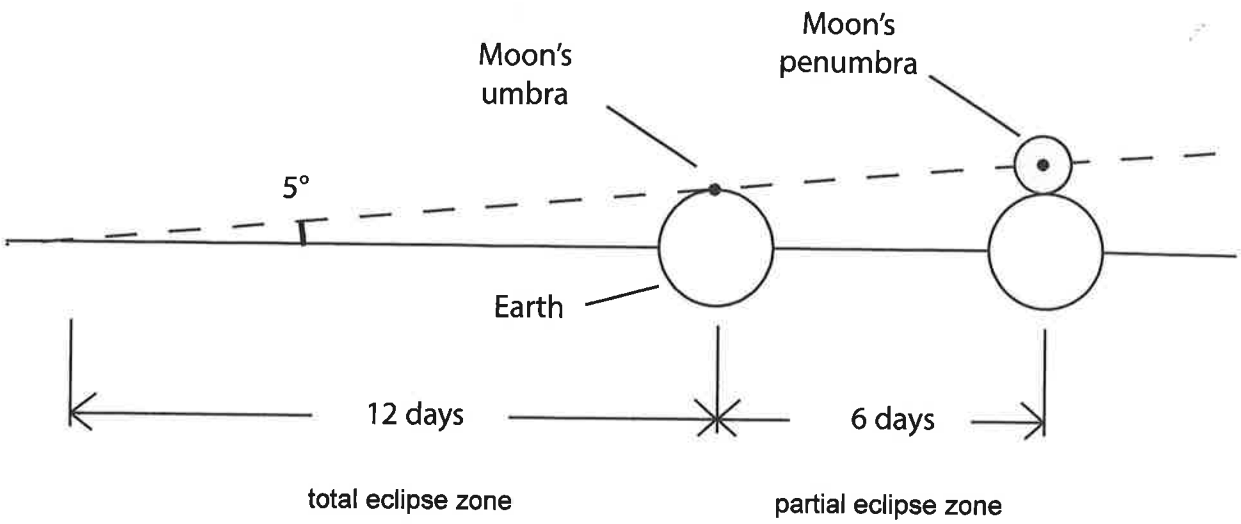


Figure 7.21: Total and partial solar eclipse zones

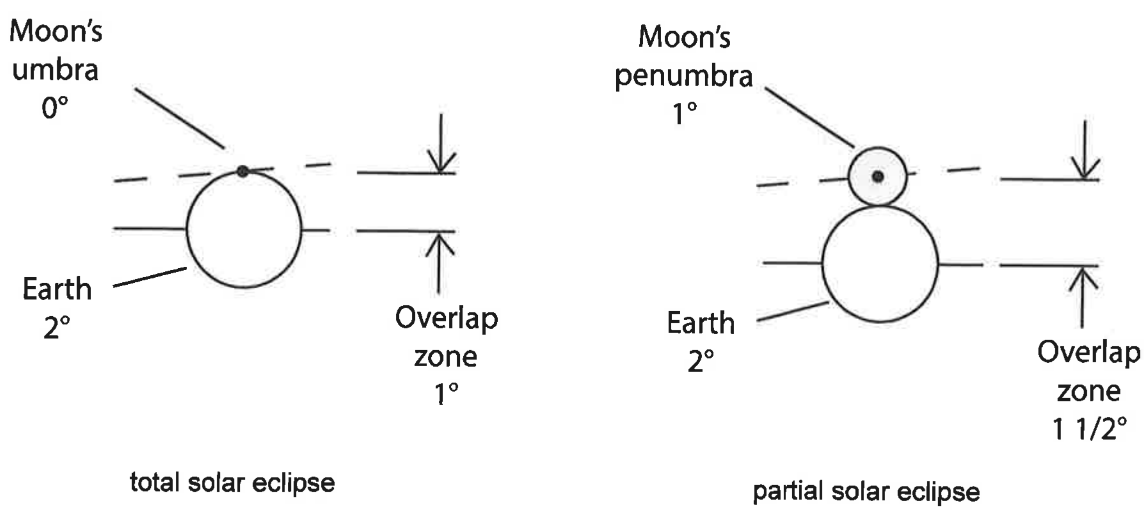


Figure 7.22: Total and partial solar eclipse, detail

If we go back and look more closely at the lunar eclipse season, we see that the 24-day season we calculated is for total and partial eclipses. How many days is the season for each? The season for total lunar eclipses is 12 days, 6 on each side of the node, as shown below and on the following page.

The season for partial eclipses is another 12 days, and the season for penumbral eclipses is another 12 days. That makes the complete lunar eclipse season 36 days.

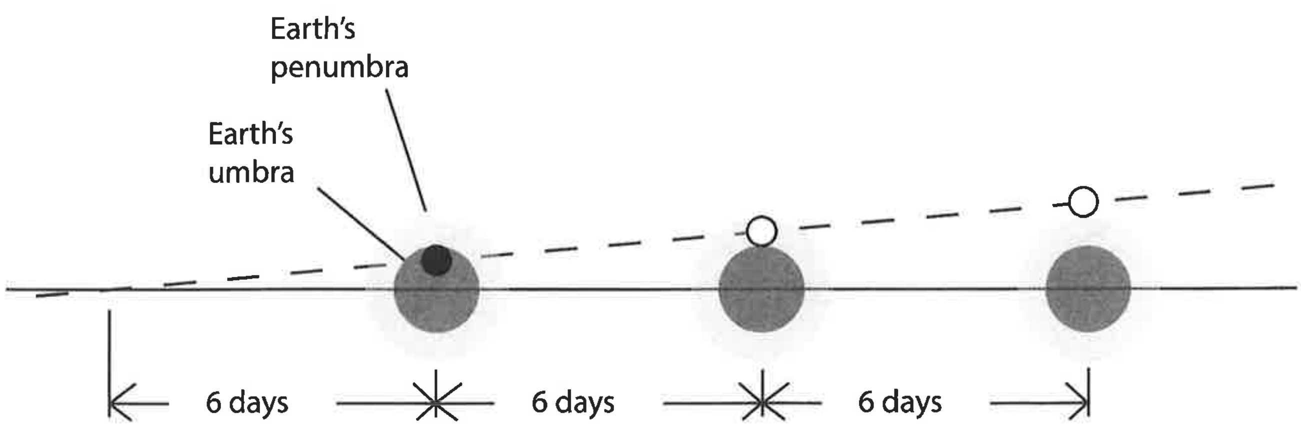


Figure 7.23: Total and partial lunar eclipse zone

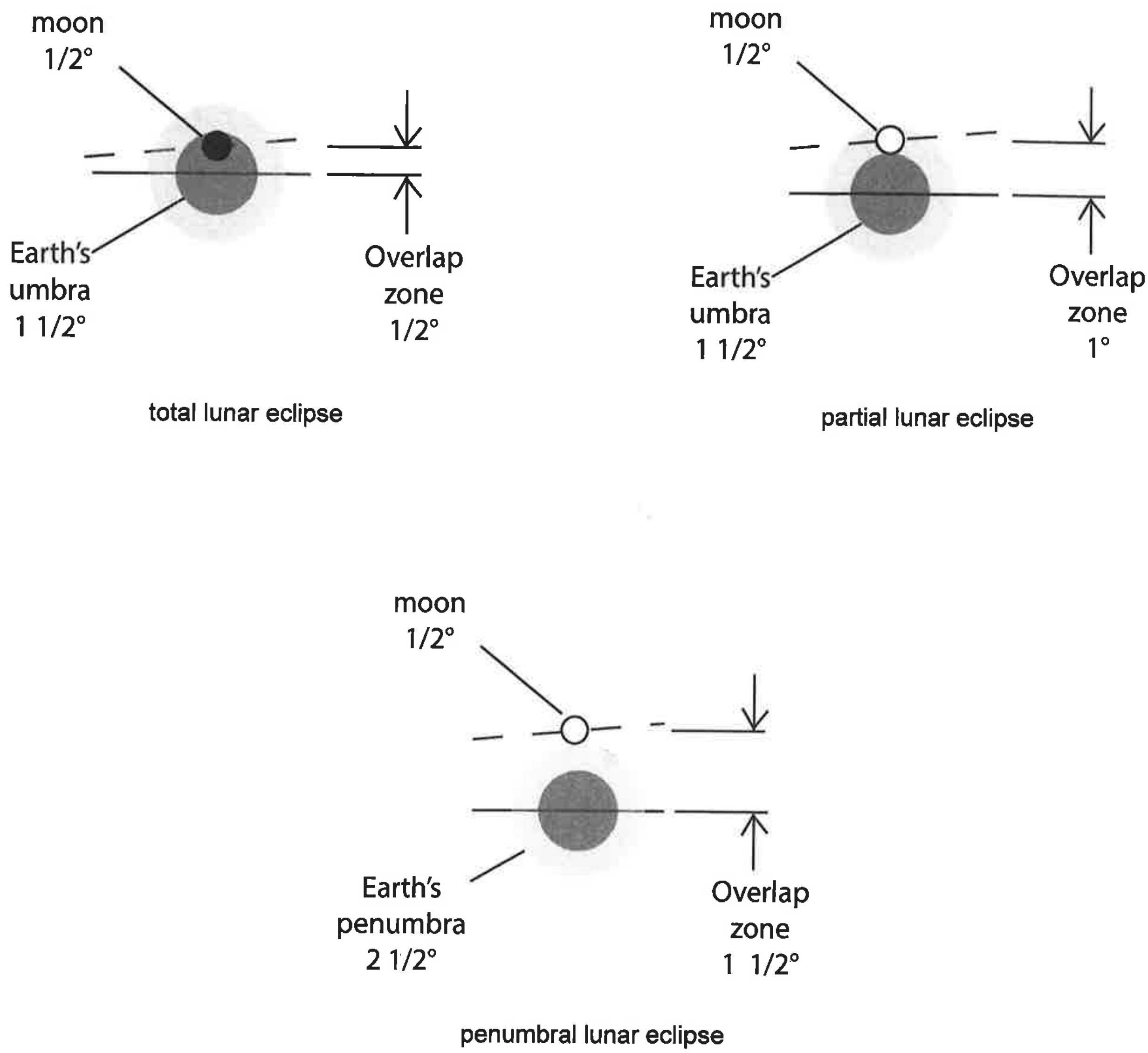


Figure 7.24: Total, partial, and penumbral lunar eclipse, detail

As the Sun travels along the ecliptic throughout the year, it crosses paths with the Moon about every 6 months, as shown in Figure 7.25. Notice the Sun is nearing the ascending node.

If a new Moon occurs while the Sun is in the zone around the node, a solar eclipse will occur. If a full Moon occurs while the Sun is in this zone, a lunar eclipse will occur in the zone near the descending node.

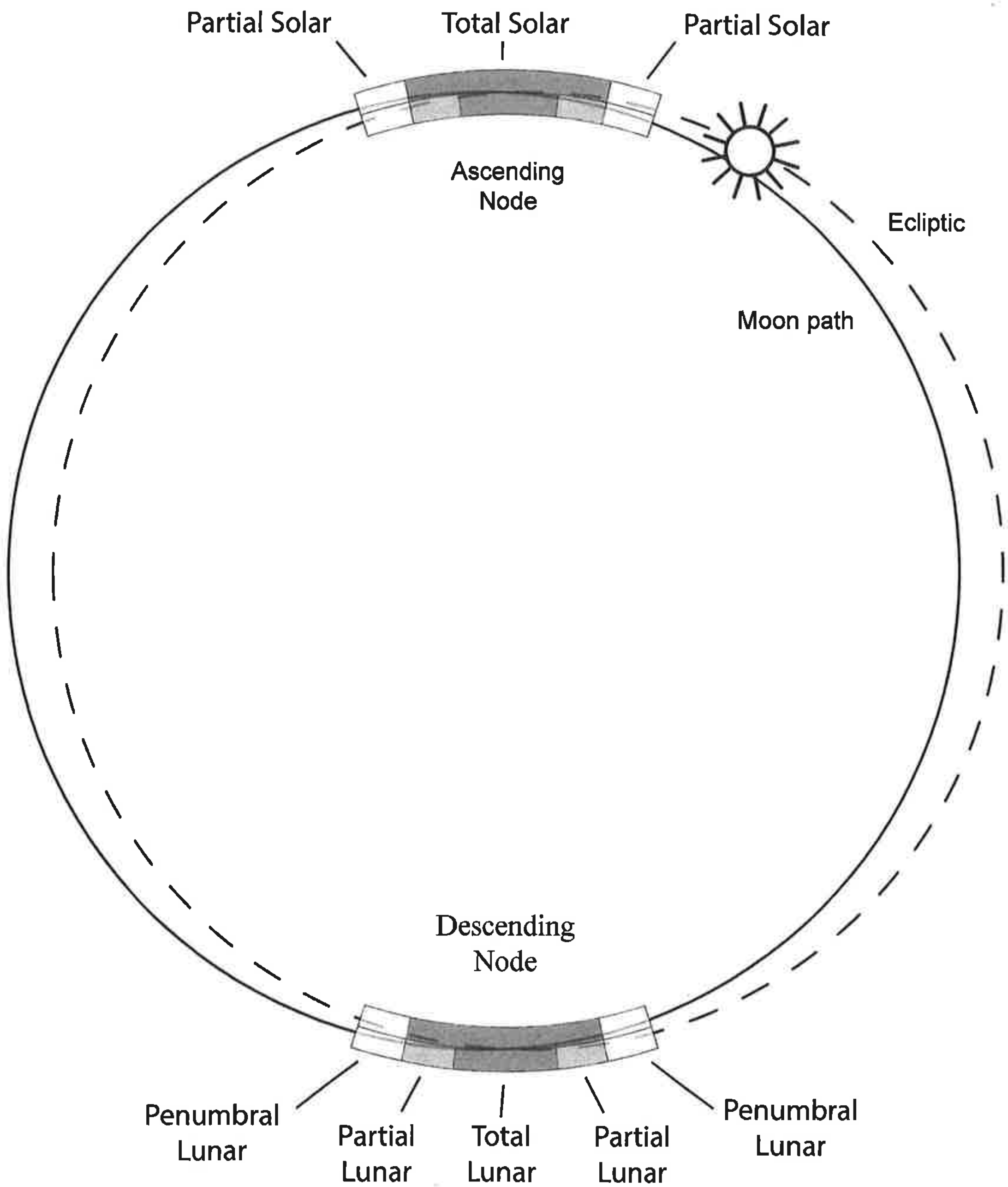


Figure 7.25: The Sun travels along the ecliptic

There are 29 ½ days between new Moons, and the partial and total solar eclipse zones are 24 and 36 days. A different combination of eclipses can occur each season.

The same is true for lunar eclipses. The total, partial and penumbral eclipse zones are 12, 24, and 36 days long, respectively. Figure 7.26 shows the possibilities for an eclipse season.

Now let's answer the question we asked a while back. How can you have solar and lunar eclipses in the same month? To answer the question, look at the new and full Moons of April - July 2012, as shown in Figure 7.27 on the following pages.

A new Moon occurs on April 21, but it is too far from a node for an eclipse. The same is true for the full Moon of May 6. But the new Moon of May 20 occurs just before the descending node, close enough for a total eclipse, the annular variety. The next full Moon, on June 4, is near enough to a node to cause a partial lunar eclipse.

By the time of the next new Moon, June 19, the Sun has already moved past the eclipse zone. The next full Moon, July 3, is also past the zone.

The next eclipse season occurs when the Sun nears the ascending node in Libra, as shown in Figure 7.28. Notice that the lunar nodes have shifted almost 10° clockwise, so the descending node is between Ram and Bull, and the ascending node is in Libra. The amount of shift between seasons is about ¾ of the width of the total lunar eclipse zone.

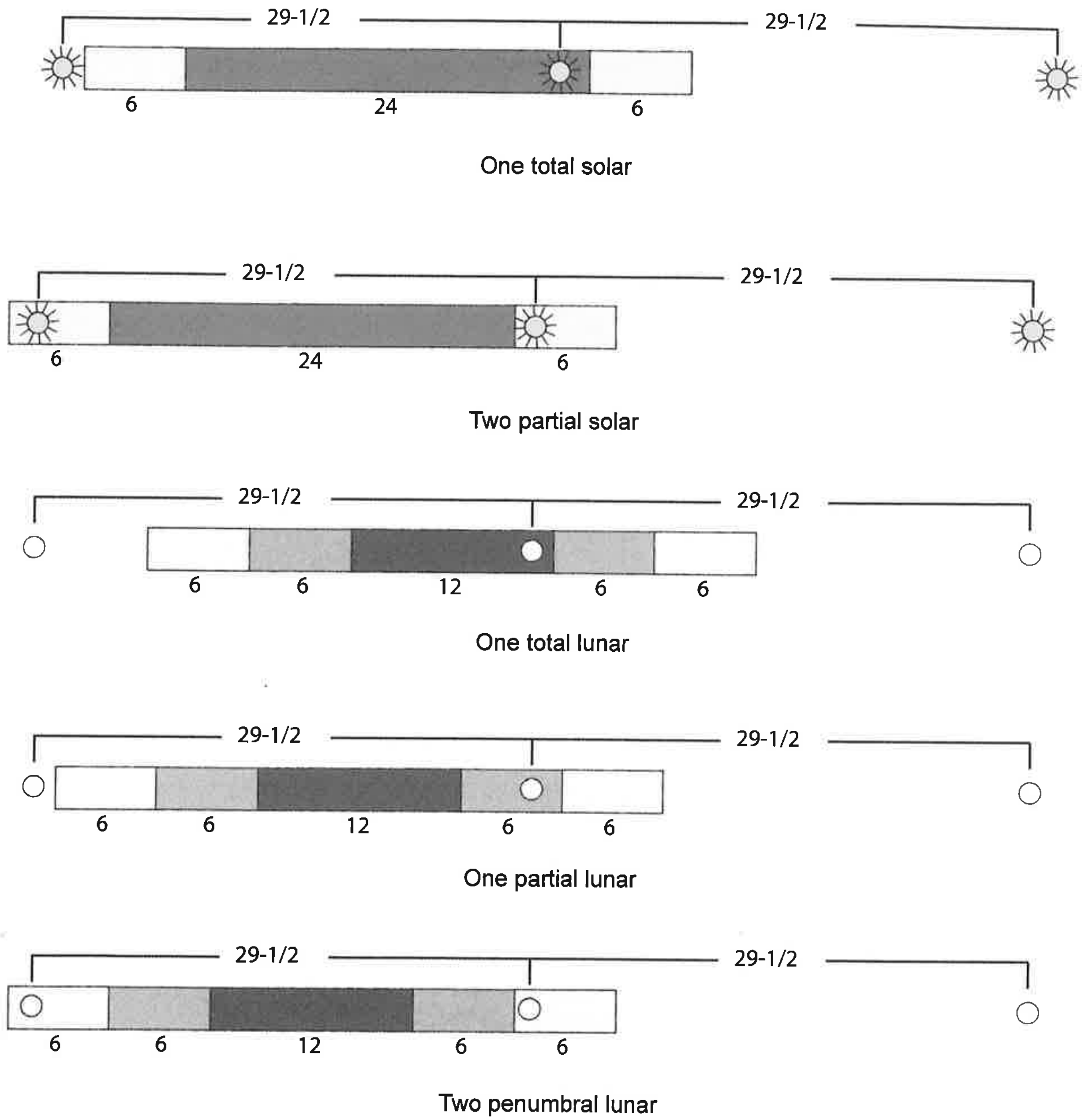


Figure 7.26: Combinations of eclipses possible in an eclipse season

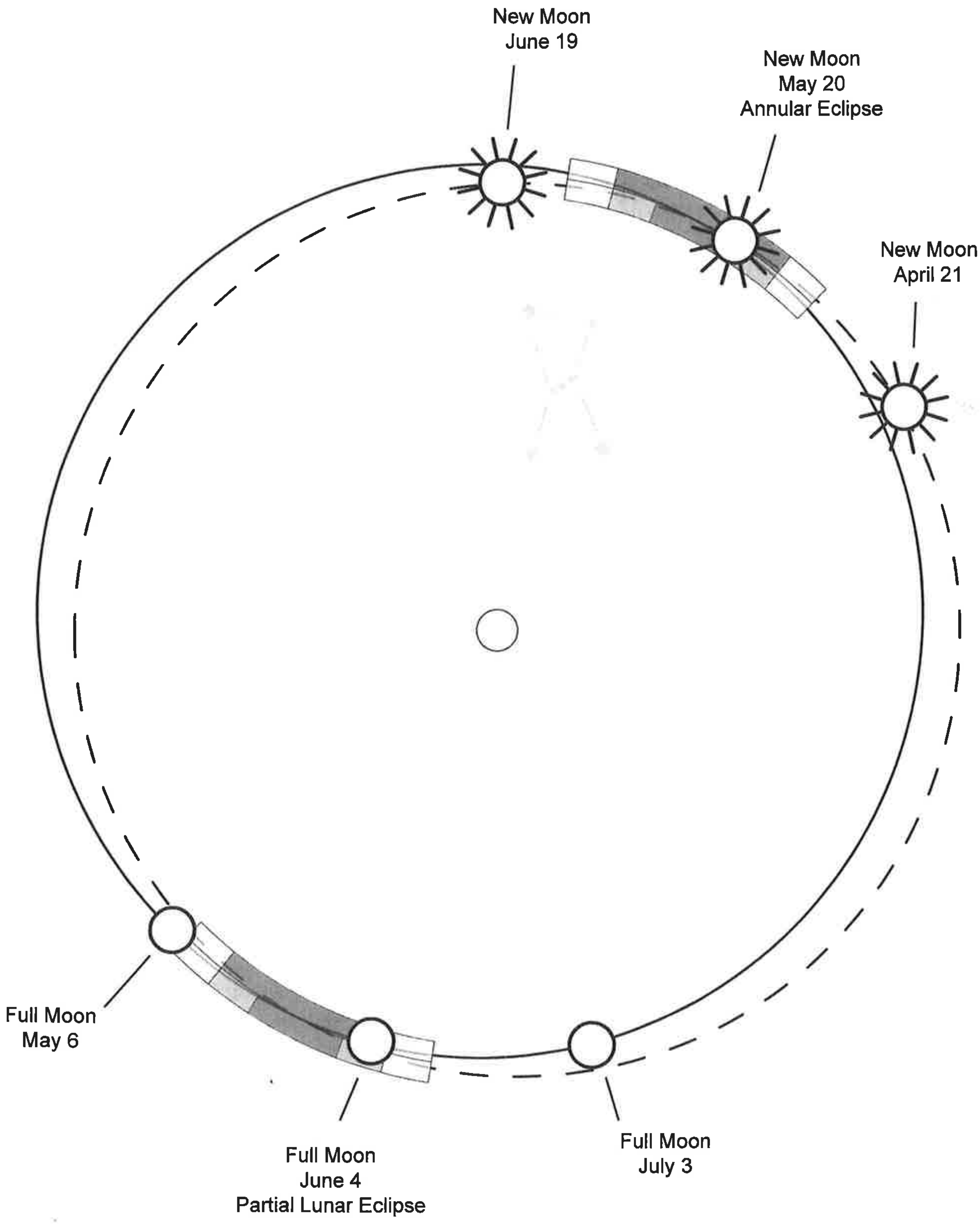


Figure 7.27: April-July 2012 saw solar and lunar eclipses in the same month

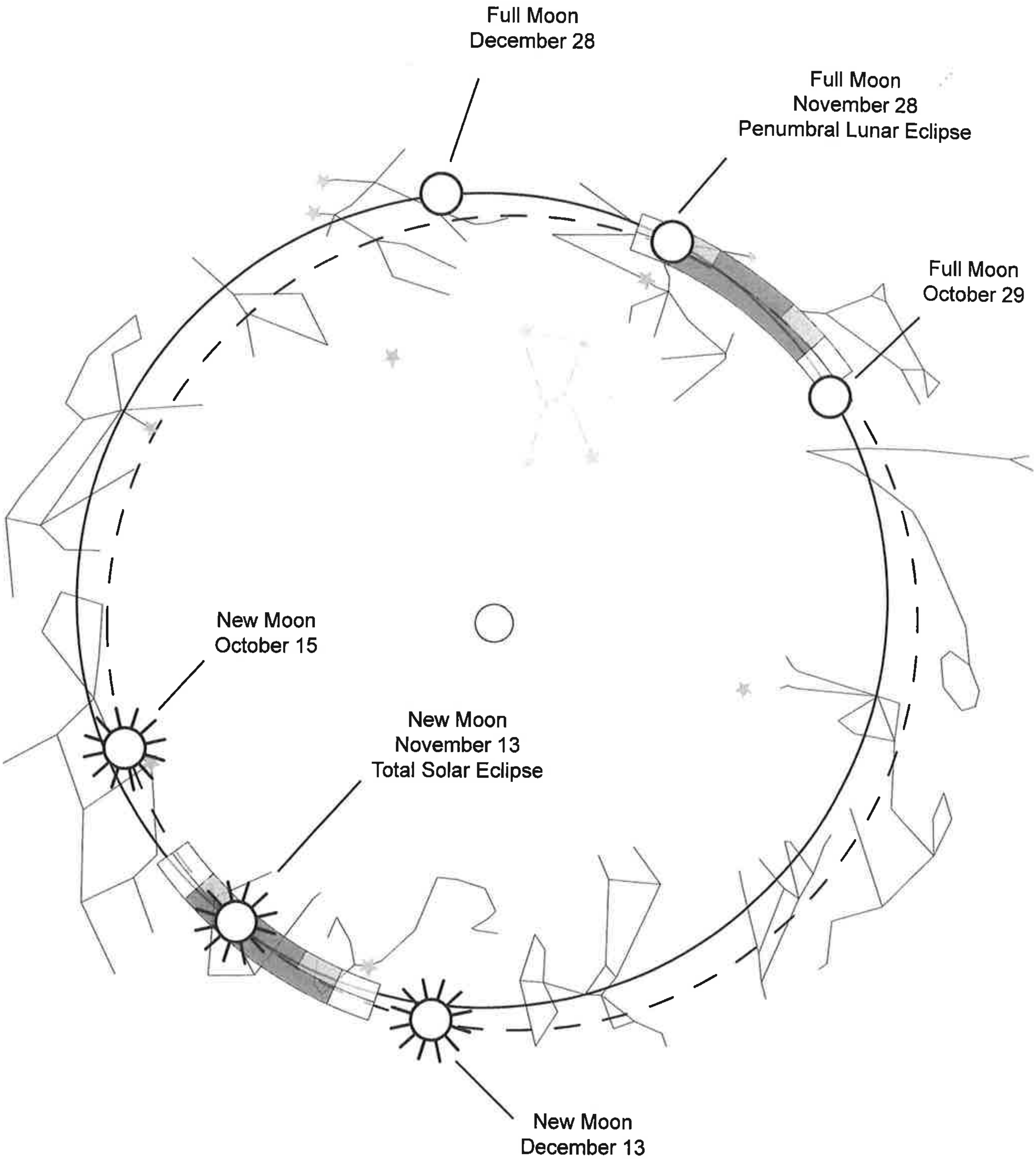


Figure 7.28: Next eclipse season, Sept-Dec 2012

Given the frequency of eclipses, why do they have a reputation for being such rare events?

The reason is that from any particular place on Earth, only some of the eclipses are visible. When a lunar eclipse occurs, it can be seen from half the places on Earth, the half that is facing the Moon as shown in Figure 7.29. The other half faces the Sun so the eclipse happens out of view.

Solar eclipses are even rarer to see. As with lunar eclipses, the half of Earth facing away from the eclipse is automatically out. But with Solar eclipses, even if you are on the right side of Earth during the eclipse, you still may not be in the eclipse path, as shown in Figure 7.30.

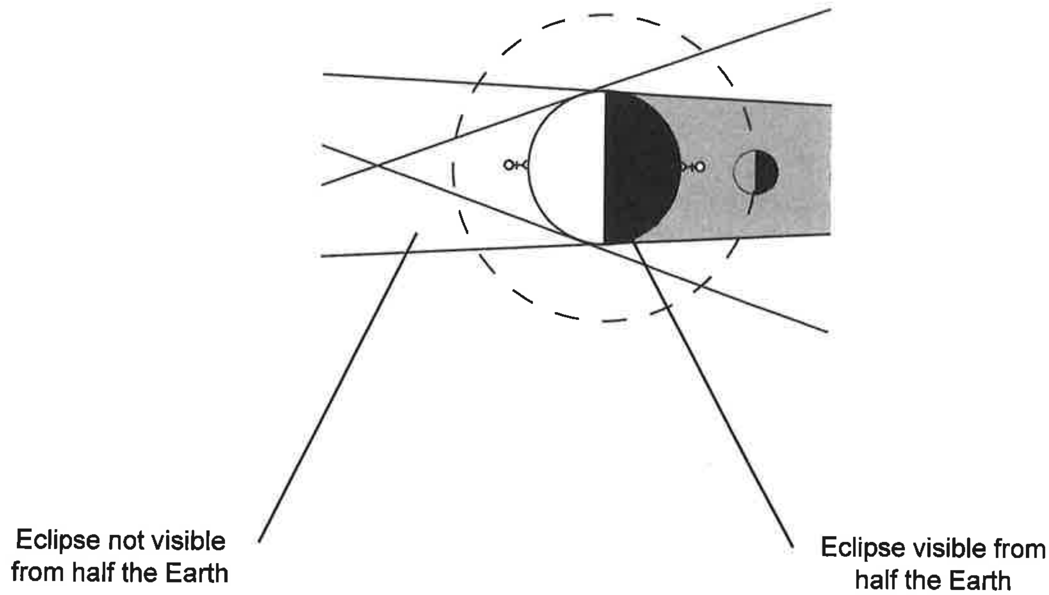


Figure 7.29: Lunar eclipse is only visible to the half of Earth facing the Moon

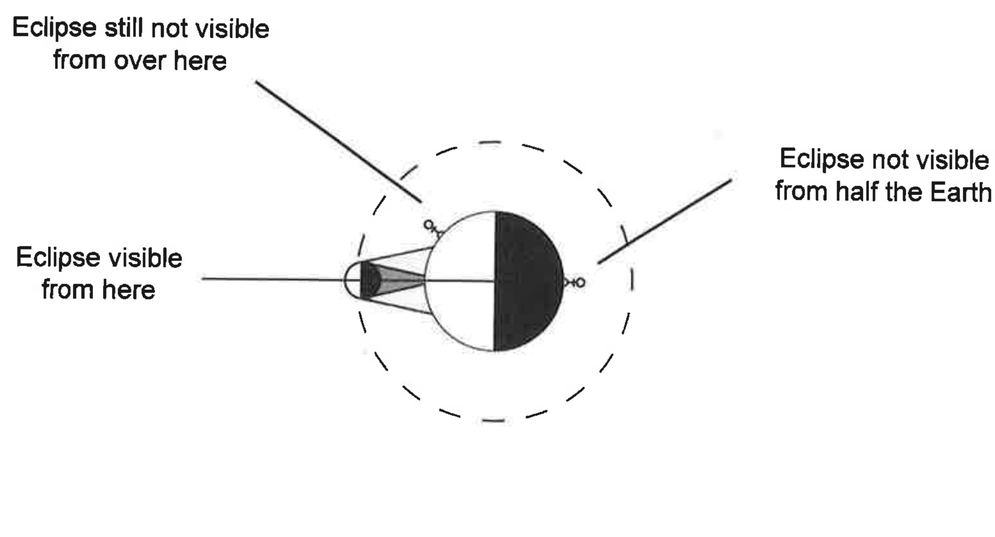


Figure 7.30: Solar eclipse is only visible to the people in the eclipse path