Key Concepts

Chapter 10

Power

Wavelength

Frequency

Planck's constant

Diffraction

Interference

Photon energy

Diffraction and Interference Patterns

A laser beam emits 10^{17} photons per second and has a power of 100 mW. The beam is incident on a single slit of width 0.2 mm. A screen is positioned 1 meter beyond the slit. What is the distance between the first minimum on the left and the first minimum on the right of the central maximum? ($h = 6.6 \times 10^{-34}$ J·s.)

1) Find the amount of energy in 1 photon.

$$100 \text{ mW} = 0.1 \text{ W} = 0.1 \text{ J/s}$$

$$\frac{0.1 \text{ J/s}}{10^{17} \text{ photons/s}} = 1 \times 10^{-18} \text{ J/photon}$$

Use dimensional analysis to find the amount of energy in one photon. Watts can be written as joules/second. Divide this by the number of photons emitted per second to find the number of joules per photon.

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2) Find the wavelength of the laser beam.

Use the energy of the photon to determine the wavelength from the formula $E = hf$. Use the relation $c = f\lambda$ to get the equation in terms of wavelength, λ.

$$E = hf = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E} = \frac{(6.6 \times 10^{-34})(3 \times 10^{8})}{(10^{-18})} = 1.98 \times 10^{-7} \text{ m}$$

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3) Find the angle to the first diffraction minima.

Use the general formula relating the angle of the diffraction minima, θ, to the slit width, a, and wavelength, λ. For the first minima, $m = 1$.

generic: $a \sin \theta = m\lambda$, $m = 1, 2, 3, \ldots$

$a = 0.2 \times 10^{-3}$ m

$m = 1$

$(0.2 \times 10^{-3}) \sin \theta_1 = (1)(1.98 \times 10^{-7})$

$\sin \theta_1 = 9.9 \times 10^{-4}$

When angles are extremely small, $\sin \theta \approx \tan \theta \approx \theta$, in radians.

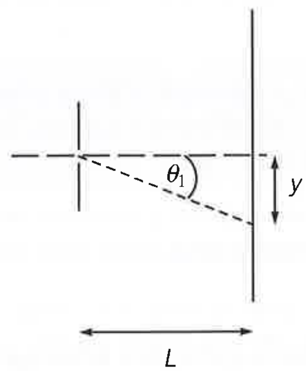
$$\theta_1 \approx 9.9 \times 10^{-4}$$

Takeaways

Any problem involving light can feature questions about power, wavelength, frequency, and energy. Solving this problem requires an understanding of all of these topics before you can get to the diffraction step.

In most diffraction problems, you will use the small-angle approximation for sin (θ) and tan (θ).

4) Find the distance between the minima.



From the diagram, we can see that the angle θ_1 to the first minimum is related by trigonometry to the distance to the minimum along the screen, y, and the distance from the screen to the slit, L. Once again, $\tan\theta \approx \theta$ because the angle is very small. The distance between the two minima equals $2y$. Solve for $2y$.

$$\tan\theta_1 = \frac{y}{L}$$
$$\theta_1 \approx \frac{y}{L}$$
$$y = L\theta_1$$
$$d = 2y = 2L\theta_1 = 2(1)(9.9 \times 10^{-4}) = 1.98 \times 10^{-3}$$
$$d = 1.98 \text{ mm}$$

Similar Questions

1) At what angle is the third diffraction minimum for a beam of 632.8 nm wavelength light incident on a single slit of width 10 μm?
2) What is the angular separation between the first two minima for a double-slit interference pattern when the slit separation is 0.1 mm and the wavelength of the light is 550.1 nm?
3) A single-slit interference pattern has an angle of 1×10^{-5} radians to the first interference minimum. If the wavelength of the light is known to be 714.5 nm, what is the width of the slit?

Things to Watch Out For

It is important to understand the formulas for diffraction patterns for single and double slits, because they look similar but have slightly different meanings.