Key Concepts

Chapter 9

Standing waves

Frequency

Wavelength

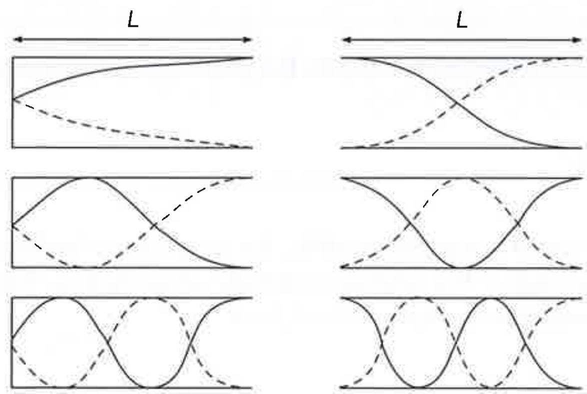
Takeaways

It is important to understand fully the meanings of the formulas describing standing waves. Make sure that you know how to derive them on a blank piece of paper. This will solidify your understanding. Moreover, it is possible to ask questions about standing waves that test your understanding of what the different harmonics look like, such as in the diagram in step 1. Be sure to understand the visual pattern.

Standing Waves

When the harmonics of a certain pipe are listed in order of increasing frequency, the second harmonic and third harmonics in the list are at 420 Hz and 700 Hz respectively. Is this pipe open at both ends, or is it closed at one end and open at the other? What is the length of the pipe? (The speed of sound in air at 0°C is 331 m/s.)

1) Identify the pattern in resonant wavelengths in each of the pipe types.



Open at one end   Open on both ends

One end open: $\lambda_5 = \frac{3}{5}\lambda_3$

Both ends open: $\lambda_3 = \frac{2}{3}\lambda_2$

To tell which type of pipe this is, you need to know or be able to derive the formula for the frequency of each type of pipe. The closed end of the pipe must have a node, whereas the open ends must have antinodes. This means that only certain wavelengths are allowed to resonate.

The pipe that is open on one end is shown on the left. For the three harmonics, or modes, shown, the length of the pipe is $\frac{1}{4}\lambda$, $\frac{3}{4}\lambda$, and $\frac{5}{4}\lambda$. This means that the wavelengths for these modes equal $4L$, $\frac{4}{3}L$, and $\frac{4}{5}L$. The general formula is $\lambda_n = \frac{4L}{n}$, where n = 1, 3, 5, and so on. Thus, $\lambda_5 = \frac{3}{5}\lambda_3$.

The pipe that is open on both ends is shown on the right. For the three modes shown, the length of the pipe is $\frac{1}{2}\lambda$, λ, and $\frac{3}{2}\lambda$. This means that the wavelengths for these modes equal $2L$, L, and $\frac{2}{3}L$. The general formula is $\lambda_n = \frac{2L}{n}$, where n = 1, 2, 3, etc. Thus $\lambda_3 = \frac{2}{3}\lambda_2$.

***Remember:** If we list the harmonics of a pipe in order of increasing frequency, then in the formula for a pipe open at one end,* n = *1 corresponds to the first harmonic in the list,* n = *3 corresponds to the second harmonic in the list,* n = *5 corresponds to the third harmonic in the list, and so on.*

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2) Find the resonant frequencies in each of the pipe types.

$$v = f\lambda \rightarrow \lambda = \frac{v}{f}$$

One end open:

$$\frac{v}{f_5} = \left(\frac{3}{5}\right)\left(\frac{v}{f_3}\right)$$

$$f_5 = \frac{5}{3} f_3$$

Both ends open:

$$\frac{v}{f_3} = \left(\frac{2}{3}\right)\left(\frac{v}{f_2}\right)$$

$$f_3 = \frac{3}{2} f_2$$

Use the relationship between velocity, frequency, and wavelength, $v = f\lambda$, to solve the expressions from step 1 for frequency.

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3) Find the ratio of the harmonic frequencies and determine the type of pipe.

$$\frac{f_{\text{3rd in list}}}{f_{\text{2nd in list}}} = \frac{700}{420} = \frac{5}{3} \rightarrow \text{pipe is open at one end.}$$

The ratio between the harmonics given in the question is $\frac{5}{3}$. From the result in step 2, this means that the pipe is open at one end.

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4) Find the length of the pipe from the wavelength expression.

$$\lambda_3 = \frac{v}{f_3} = \frac{321}{420} = 0.79\text{ m}$$

$$\lambda_3 = \frac{4L}{3} \rightarrow L = \frac{3}{4}\lambda_3 = \left(\frac{3}{4}\right)(0.79) = 0.59\text{ m}$$

Use the formula $v = f\lambda$ once again to solve for the wavelength of the second harmonic in the list. Then use the expression for wavelength from part 1 to solve for the length of the pipe. Remember that for a pipe open at one end, when we list the harmonics in order of increasing frequency, $n = 3$ for the second harmonic in the list.

Similar Questions

1) Where are the nodes located along a pipe, open at both ends, that is resonating at its third harmonic?
2) What is the wavelength of the fourth harmonic of a vibrating string with a length of 30 cm?
3) If we list the harmonics of a certain pipe in order of increasing frequency, the nodes of the third harmonic in the list are located at 40 cm intervals. What is the length of this pipe if one end of the pipe is closed?