## Key Concepts

Chapter 9

Dimensional analysis

Harmonic motion

Friction

Angular frequency $\omega = 2\pi f$ ($s^{-1}$)

Springs

## Takeaways

Use dimensional analysis to check your work. Get comfortable with algebraic manipulations.

## Things to Watch Out For

Don't be thrown by situations that seem novel or overly complex. Nearly all of the information you need to solve this problem is given explicitly. Keep your units straight! In this example, both the spring constant and the damping constant have the same scalar value, but their units are different.

# Dampened Harmonic Motion

A vertical spring-mass system is submerged in oil. The spring has a stiffness coefficient of 3 N/m, and the mass is 1 kg. If the dampened angular frequency $\omega_d$ of oscillation is 0.866 Hz, find the damping coefficient $b$ for the oil and how long it takes the spring to reach 50 percent of its original amplitude. The equations for dampened angular frequency and amplitude are provided below.

$$\omega_d = \sqrt{\omega^2 - \left(\frac{b}{2m}\right)^2}$$

$$A = A_0 e^{-\left(\frac{b}{2m}\right)t}$$

**1) Find the angular frequency of the spring.**

$$\omega = 2\pi f = \sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{\frac{3}{1}} \approx 1.73 \text{ Hz}$$

If the spring-mass system were free of nonconservative forces, it would oscillate indefinitely with an angular frequency of $\omega$. Looking at the equation for dampened frequency $\omega_d$, we see that we're basically adjusting $\omega$ for friction by subtracting a quantity specific to the damping coefficient and mass.

***Remember:*** *Use dimensional analysis whenever possible to check your work. In this case, we know that the frequency of oscillation needs to be in Hz or $s^{-1}$. The spring constant* k *is in N/m and the mass* m *is in kg. Even if we forget whether* k *goes on top or on the bottom in this equation, thinking critically and using dimensional analysis allows us to find the right equation. If you are uncomfortable using the unit of newton, think back to Newton's second law:* F = ma. *A newton is simply kg(m/$s^2$).*

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**2) Use the first equation to find the damping coefficient.**

$$\omega_d = \sqrt{\omega^2 - \left(\frac{b}{2m}\right)^2}$$

Here we are being tested on algebra. Square both sides of the equation to get rid of the square root on the right side. Solve for *b*. Dimensional analysis in the penultimate step shows us that we are multiplying angular frequency with mass. The individual units, respectively, are $s^{-1}$ and kg, so our product, the damping coefficient, must have that unit. (Try using dimensional analysis to prove to yourself that $b$ can also be recorded in $N \cdot s/m$.)

$$\omega_d = \sqrt{\omega^2 - \left(\frac{b}{2m}\right)^2}$$

$$0.866 = \sqrt{(1.73)^2 - \left[\frac{b}{2(1)}\right]^2} \rightarrow 0.75 \approx 3 - \left(\frac{b^2}{4}\right)$$

$$(0.75-3)4 = -b^2$$

$$b = 3 \text{ kg/s}$$

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**3) Use the second equation to see how the amplitude of oscillation diminishes with time.**

$$A = A_0 e^{-\left(\frac{b}{2m}\right)t}$$

This, too, is a test of our algebraic skills. We are asked to find the time needed to reach 50 percent of the original amplitude of oscillation. Thus, $A = 0.5A_0$. Plug into the equation and cancel the $A_0$ term.

$$0.5A_0 = A_0 e^{-\left(\frac{3}{2(1)}\right)t}$$

$$0.5 = e^{-\left(\frac{3}{2(1)}\right)t}$$

To find $t$, we need to get rid of the number $e$. Take the natural log of both sides (because $\ln(e^x) = x$) to drop $t$ into a workable domain. Then solve for $t$.

$$\ln 0.5 = -\left(\frac{3}{2(1)}\right)t$$

$$-2(-0.69) \div 3 = t$$

$$0.46 \text{ s} = t$$

Dimensional analysis of the exponent on $e$ reinforces our units are correct. The exponent overall should have no units, so to cancel out s/kg, we need $b$ to have the unit kg/s.

***Remember:*** *On the MCAT, you will not be expected to take natural logs. You should, however, be able to estimate logarithms with the base 10. Occasionally, you will be given the conversion factor between ln and log.*

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## Similar Questions

1) By what percentage does the frequency diminish in a horizontal spring-mass system where $k = 1 \times 10^3$ N/m and $m = 4$ kg if the motion is dampened by a frictional force that has a damping coefficient of 2 kg/s?
2) What is the period of oscillation of a curled filament with a mass of 100 g and a spring constant of 15 N/m? When heated to 113°C, the filament undergoes a spontaneous exothermic reaction to dissipate the heat. If the new dampening coefficient is $5 \times 10^{-3}$ kg/s, how much energy is lost in 1.5 minutes? Use the equation $E = E_0 e^{-\left(\frac{b}{m}\right)t}$.