Key Concepts

Chapter 9

Newton's laws

Tension

Weight

$F = ma$ ($N = kg \cdot m/s^2$)

Takeaways

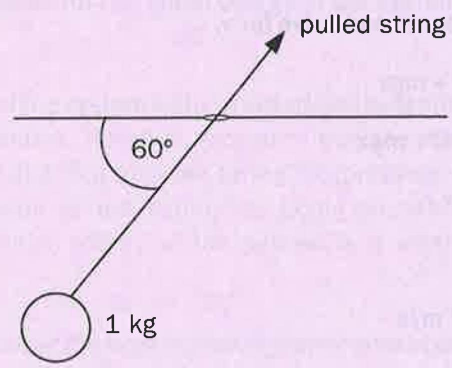
This problem seems complicated at first because it is an unusual setup, and the motion of the ball seems like it would take a very complicated path. Do not be scared off, because the same problem-solving process applies to this as to any other force/acceleration problem: (1) draw the free-body diagram, (2) add up the forces in each direction, and (3) solve.

Things to Watch Out For

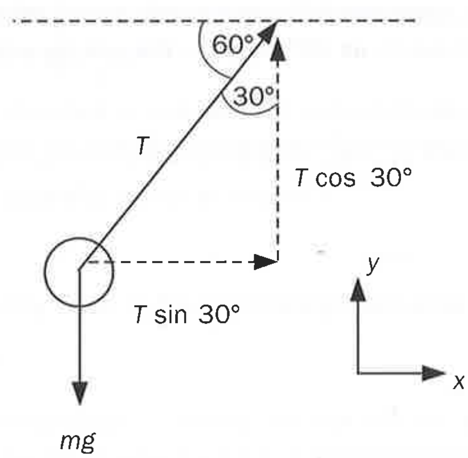
Keeping track of signs is important in these problems. If you had reversed the sign of F_y in calculating the angle, you would have gotten a positive angle instead, and those sorts of mistakes are always included among the answer choices.

Acceleration of a Pendulum

A small ball of mass 1 kg is tied to a string. The string is fed through a small hole as shown in the diagram below. If the ball is held at a 60-degree angle, as shown, and is then released from rest as the string is pulled with a constant force of 10 N at the same angle, what is the initial acceleration (magnitude and direction) of the ball?



1) Draw a free-body diagram.



There are two forces acting on the ball: the tension in the string (labeled T) and the weight of the ball (which equals mg). If we use a standard x-y coordinate plane, the tension in the string must be broken into its **X** and **Y** components using trigonometry.

2) Add the forces in the x- and y-directions.

$\Sigma F_x = ma_x = T \sin 30° = (10)(0.5) = 5\text{ N}$

$\Sigma F_y = ma_y = T \cos 30° - mg = (10)(0.866) - (1)(9.8)$

$= -1.14\text{ N}$

Add the forces in the *x*- and *y*-directions separately. The net force in a direction always equals the mass times the acceleration in that direction. This is Newton's second law.

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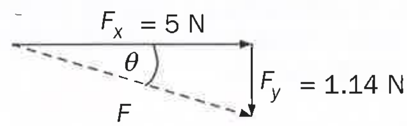
3) Find the magnitude and direction of the net force.



$$F^2 = F_x^2 + F_y^2$$

$$F = (F_x^2 + F_y^2)^{\frac{1}{2}} = [5^2 + (-1.14)^2]^{\frac{1}{2}} = 5.13\,\text{N}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{-1.14}{5}\right) = -12.8°$$

In step 2, we found the **X** and **Y** components of the net force. These two vectors form the sides of a right triangle. To find the magnitude of the net force, use the Pythagorean theorem. To find the angle of the net force, use trigonometry.

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4) Find the acceleration.

$$F_{net} = ma \rightarrow a = \frac{F_{net}}{m} = \frac{5.13}{1} = 5.13\ \text{m/s}^2$$

$$\theta = -12.8°$$

Use Newton's second law to determine the acceleration from the net force. The direction of the acceleration is the same as the direction of the net force. This is always true.

Similar Questions

1) A car drives around a circular track of radius 100 m at a speed of 120 m/s. What is the magnitude and direction of the acceleration of the car?
2) A 20 kg block is pushed from the east with a force of 100 N, from the west with a force of 20 N, and from the south with a force of 150 N. In what direction does the block travel?
3) A 10 kg sled is pulled at an angle of 35° east of north by a force of 100 N and with a force of 150 N directed due east. A friction force of 50 N acts as well. What is the acceleration (both magnitude and direction) of the sled?