Key Concepts

Chapter 9

Conservation of energy

Springs

Gravitational potential energy: $U = mgh$ (J)

Spring potential energy: $U = \frac{1}{2}kx^2$ (J)

Kinetic energy: $K = \frac{1}{2}mv^2$ (J)

Takeaways

This problem is solved in the same way as every conservation of energy problem: Write expressions for the total energy of the system at two points, set them equal, and solve for the unknown quantity.

Things to Watch Out For

Variations on this problem could include questions about work required to compress a spring, compression along an incline, or the introduction of nonconservative forces. Apply the process for solving conservation of energy problems, and you will succeed!

Energy and Springs

A block of mass 2 kg falls from a height of 3 meters onto a spring, and the spring reaches a maximum compression of 20 cm. What is the spring constant for this spring? When the block bounces off of the spring, how fast is it going?

1) Write an expression for the initial energy of the system.

$$E_i = mg(h + x)$$

The initial energy of the system is the gravitational potential energy of the block at a height of 3 meters. However, recognize that the block will fall 3 meters plus an additional distance from the spring compressing. Let $h = 3$ meters and x = the compression of the spring. By doing so, we are saying that the gravitational potential energy of the system is 0 when the spring is fully compressed.

Remember: *You can set the potential energy to be zero at whatever point is most convenient, but you must be consistent through the entire problem.*

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2) Write an expression for the final energy of the system.

$$\text{spring } U = \frac{1}{2}kx^2 \rightarrow E_f = \frac{1}{2}kx^2$$

When a spring is stretched or compressed, potential energy is stored in the spring. This energy is given by the formula $U = \frac{1}{2}kx^2$. As we saw in step 1, there is no gravitational potential energy at this point.

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3) Set the final energy equal to the initial energy and solve for *k*.

$$mg(h + x) = \frac{1}{2}kx^2$$

Because of the conservation of energy, we can set the total energy of the system at any two points equal to each other. Set the initial energy equal to the final energy and solve for k.

$$mgh + mgx = \frac{1}{2}kx^2$$
$$(2)(9.8)(3) + 2(9.8)(0.2) = \frac{1}{2}(k)(0.2)^2$$
$$58.8 + 3.92 = 0.02k$$
$$k = 3{,}136 \text{ N/m}$$

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4) Write an expression for energy of the system at the point that the block bounces off of the spring.

$$E_3 = \frac{1}{2}mv^2 + mgx$$

As the block bounces off of the spring, it has kinetic energy equal to $\frac{1}{2}mv^2$. It also has potential energy equal to mgx, because it is now at a height of x. The total energy is the sum of these two amounts.

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5) Set two energies equal and solve for *v*.

$$mg(h + x) = \frac{1}{2}mv^2 + mgx$$

$$mgh + mgx = \frac{1}{2}mv^2 + mgx$$

$$gh + gx = \frac{1}{2}v^2 + gx$$

$$gh = \frac{1}{2}v^2$$

$$v = (2gh)^{\frac{1}{2}} = 7.7 \text{ m/s}$$

Much as in step 3, we can set the total energy of the system at any two points equal to each other. The numbers look easier to deal with for the energy expression from step 1, so choosing this one will save some time. (Note that the velocity is the same when it first hits the spring as when it rebounds off of the spring. There is no difference in the energy expression for these two points.)

Similar Questions

1) A spring is compressed vertically 10 cm, and a 5 kg block is placed on top of it. The spring has a spring constant of 50 N/m. If the system is released from rest, what is the maximum height achieved by the block?
2) A spring has a spring constant of 100 N/cm. How much work is required to alter the spring from a compression of 10 cm to a compression of 50 cm?
3) A 5 kg mass and a 2 kg mass are placed on two identical springs (k = 1.3 kN/m). What is the ratio of the maximum compressions of these two springs?