Key Concepts

Chapter 8

Ohm's law: $V = IR$

Resistors in series: $R_{series} = R_1 + R_2$

Resistors in parallel:

$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2}$$

Takeaways

There is a standard process for solving these types of resistor problems:

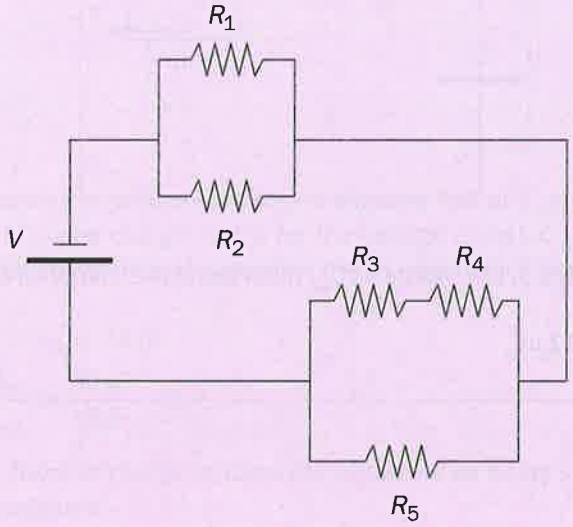
1) Find the equivalent resistance by combining series and parallel resistors.
2) Find the current through the circuit.
3) Expand the circuit back out, step-by-step.
4) Apply knowledge of series and parallel resistors to find the quantity of interest.

Things to Watch Out For

The arithmetic for calculating the equivalent resistance in parallel can be confusing. Another version of the formula is $R_{parallel} = \frac{R_1 R_2}{R_1 + R_2}$. Note, however, that this only works for two resistors and cannot be expanded simply to account for three resistors.

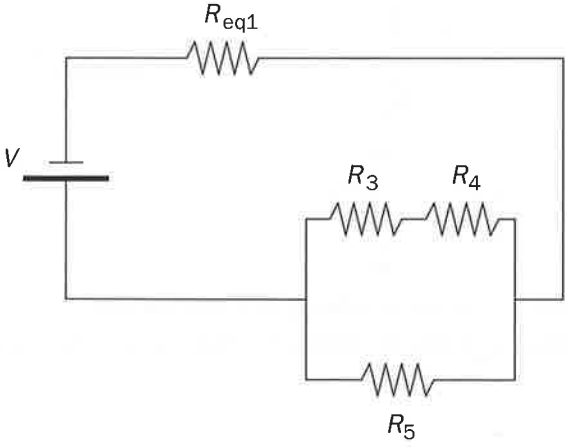
Resistor Circuits

What is the current through R_1 in the circuit shown below?
($R_1 = 30\ \Omega$; $R_2 = 6\ \Omega$; $R_3 = 20\ \Omega$; $R_4 = 10\ \Omega$; $R_5 = 30\ \Omega$; $V = 30$ V)



1) Find the equivalent resistance of the network.

The first part of this problem is to find the equivalent resistance of the entire circuit. This will take several steps. Begin by combining R_1 and R_2 using the equation for the equivalent of two parallel resistors.

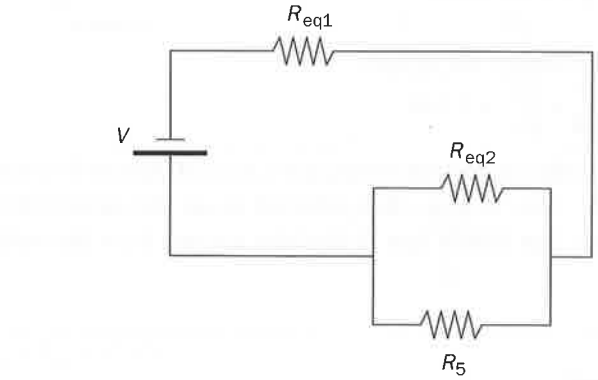


$$\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq1}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{30} + \frac{1}{6} = \frac{1}{30} + \frac{5}{30} = \frac{6}{30}$$

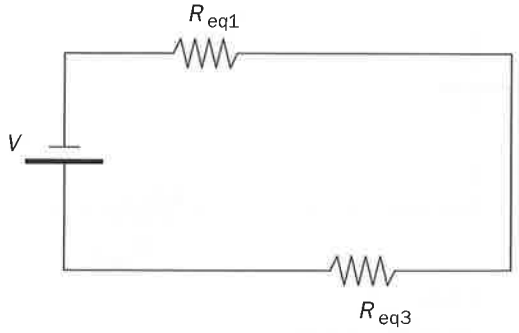
$$R_{eq1} = \frac{30}{6} = 5\ \Omega$$

2) Find the equivalent resistance of the network.
Combine R_3 and R_4, which are in series. Resistors in series add simply.



generic: $R_{series} = R_1 + R_2$
$R_{eq2} = R_3 + R_4 = 20 + 10 = 30\ \Omega$

3) Find the equivalent resistance of the network.
Combine the equivalent resistance from step 2, R_{eq2}, with R_5. These resistors are in parallel.



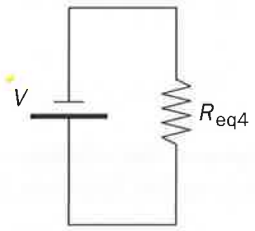
$$\frac{1}{R_{eq3}} = \frac{1}{R_5} + \frac{1}{R_{eq2}} = \frac{1}{30} + \frac{1}{30} = \frac{2}{30}$$

$$R_{eq3} = 15\ \Omega$$

Similar Questions

1) In the circuit shown in the original question, what is the voltage across R_4?
2) Four resistors are attached in parallel to a power supply of 9 V. If the resistances of the resistors are 10 Ω, 20 Ω, 30 Ω, and 40 Ω, what is the current through the battery?
3) There are six resistors in a circuit. R_1, R_2, R_3, R_4, and R_5 are all in parallel with each other, and they are all in series with R_6. If the current leaving the battery is 10 A and all of the resistors have a resistance of 1 Ω except for R_6, which is unknown, what is the resistance of R_6 if $V = 20$ V?

4) Find the equivalent resistance of the network.
Combine the equivalent resistance from step 3, R_{eq3}, with that from step 1, R_{eq1}. These resistors are in series.



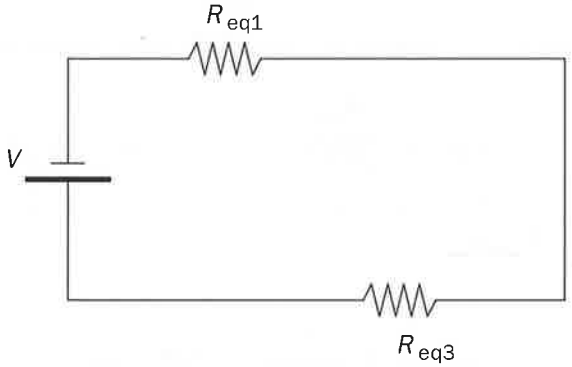
$R_{eq4} = R_{eq1} + R_{eq3} = 5 + 15 = 20\ \Omega$

5) Find the current through the circuit.

$$V = IR \rightarrow I = \frac{V}{R} = \frac{30}{20} = 1.5\ \text{A}$$

The point of finding the equivalent resistance is so that we can find the current through the circuit. This is also often referred to as the current through or leaving the battery. Use Ohm's law to find the current from the voltage and resistance.

6) Expand the circuit.
Now expand the circuit back out and apply what we know about resistors in series and parallel to find the current and voltage through individual resistors. All resistors in series must have the same current, so we know that the current from step 5 must equal the current through R_{eq1} and R_{eq3}. Use Ohm's law to find the voltage across R_{eq1}.

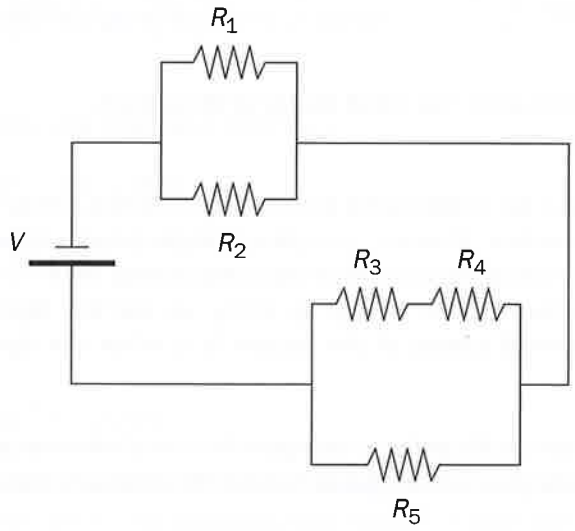


$I_{eq1} = I_{eq3} = I = 1.5\ \text{A}$
$V_{eq1} = I_{eq1}R_{eq1} = (1.5)(5) = 7.5\ \text{V}$

Remember: *Any number of resistors in series have the same current as one another.*

7) Expand the circuit again.

Because R_{eq1} is a parallel combination of R_1 and R_2, we know that R_1 and R_2 must have the same voltage as R_{eq1}. Any two (or more) circuit components in parallel must have the same voltage as the others. Use the voltage and resistance of R_1 to find the current through R_1.



$$V_{R1} = V_{eq1} = 7.5 \text{ V}$$

$$I_1 \frac{V_1}{R_1} = \frac{7.5}{30} = 0.25 \text{ A}$$

Remember: *All components attached in parallel have the same voltage as one another.*