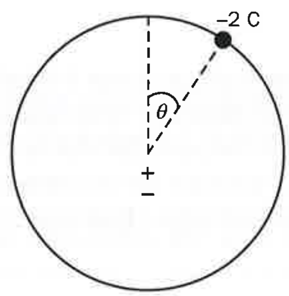
Voltage and Energy

A dipole sits at the center of a large wire circle of radius 50 cm that is sitting horizontally in a plane. A small, –2 C charge with a mass of 4 kg is constrained to slide with no friction along the loop. The potential of the dipole is given as $V = V_o \cos \theta$, where V_o is 5 and θ measures the angle from the vertical. The point charge's initial position on the loop is directly above the dipole ($\theta = 0$) of the loop and is given an initial speed of 3 m/s. How fast is the point charge moving at the point corresponding to $\theta = 90°$?



1) Write an expression for the initial energy of the system.

$$E_i = U_i + K_i$$

$$U_i = qV_i = qV_o \cos \theta_i$$

$$K_i = \frac{1}{2}mv_i^2$$

$$E = qV_o \cos \theta_i + \frac{1}{2}mv_i^2$$

The energy of the system is the sum of the potential energy and the kinetic energy. The potential energy is given by $U = qV$, where the formula for V is given in the question.

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2) Write an expression for the final energy of the system.

$$E_f = U_f + K_f$$

$$U_f = qV_f = qV_o \cos \theta_f$$

$$E = qV_o \cos \theta_f + \frac{1}{2}mv_f^2$$

Much as in step 1, the energy of the system is the sum of the potential energy and the kinetic energy.

Key Concepts

Chapter 6

Voltage

Mechanical energy

$U = qV$ (J = C·V)

$E = U + K$

$K = \frac{1}{2}mv^2$

Takeaways

This problem appears complex due to the unusual situation, but it is solved the same way as any other energy problem: (1) Write expressions for the total energy of the system at two points, (2) set the expressions equal, and (3) solve.

Things to Watch Out For

Many students do not realize that problems involving charged particles and voltages can be solved most easily using the conservation of energy equation.

3) Set the energy expressions equal to each other and solve.
Due to the conservation of energy, we can set the initial and final energies as equal. This allows us to solve for v_f. Plug in the angles, mass, charge, and initial velocity to find v_f.

$$E_i = E_f$$

$$qV_0 \cos \theta_i + \frac{1}{2}mv_i^2 = qV_0 \cos \theta_f + \frac{1}{2}mv_f^2$$

$$\theta_i = 0 \rightarrow \cos \theta_i = 1$$

$$\theta_f = 90° \rightarrow \cos \theta_f = 0$$

$$qV_0 + \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2$$

$$(-2)(5) + \frac{1}{2}(4)(3)^2 = \frac{1}{2}(4)v_f^2$$

$$8 = 2v_f^2$$

$$v_f = 2 \text{ m/s}$$

Similar Questions

1) How fast does a 1 kg ball move after falling from a height of 10 meters if the ball is thrown down with a speed 2 m/sec?
2) An alpha particle, starting from rest, travels through a potential difference of 200 V. What is the final speed of the particle? ($e = 1.6 \times 10^{-19}$ C)
3) Two protons (mass = 1.66×10^{-27} kg) initially are at rest at a distance of 10 nm from each other. They are released and accelerate away from each other. How fast are they both going after they are very far apart? ($k = 9 \times 10^9$ Nm^2/C^2)