Key Concepts

Chapter 6

Work: $W = q\Delta V$

Electrostatics

Electric potential energy, U

Takeaways

To find the work required to assemble a distribution of charges, find the work required to place each charge individually and then add. The work to place the first charge is always zero.

Things to Watch Out For

It is common to make sign errors on these types of problems. Keep this in mind when checking your work: Like charges increase in potential energy as they are brought closer; unlike charges decrease in potential energy as they are brought together.

Charge Distribution and Work

Three charges are lined up along the x-axis. Charge 1 has a charge of $+1\ \mu C$. Charge 2 has a charge of $-2\ \mu C$. Charge 3 has a charge of $+4\ \mu C$. The charges are all 1 mm away from each other. How much work was required to assemble this distribution of charge, assuming that the charges were initially very far apart? ($k = 9 \times 10^9\ N \cdot m^2/C^2$)

1) Find the work required to place charge 1.

$$W = \Delta U = q\Delta V$$
$$W = 0$$

The work done to move a charge equals the charge times the change in electric potential. The work done to place charge 1 is zero because there is no change in electric potential.

Remember: *As a matter of convention, the work to place the first charge is always zero.*

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2) Find the work required to place charge 2.

$$W = \Delta U = q\Delta V = q_2(V_f - V_i)$$
$$V_i = 0$$
$$V_f = \left(\frac{kq_1}{r_{12}}\right)$$
$$\text{generic: } V = \frac{kq}{r}$$
$$W = q_2\left[\left(\frac{kq_1}{r_{12}}\right) - 0\right]$$
$$W = (-2 \times 10^{-6})\left[\frac{(9 \times 10^9)(1 \times 10^{-6})}{1 \times 10^{-3}}\right] = -18\ J$$

The same formula is used in this step as in step 1. Here, charge 2 has an initial electric potential of zero because it is very far away from charge 1. The final electric potential is given by the formula $V = \frac{kq}{r}$, where q is the charge of the stationary charge and r_{12} is the distance between charges 1 and 2.

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3) Find the work required to place charge 3.

$$W = \Delta U = q\Delta V = q_3(V_f - V_i)$$

$$V_i = 0$$

$$V_f = \left(\frac{kq_1}{r_{13}}\right) + \left(\frac{kq_2}{r_{23}}\right)$$

$$W = q_3\left[\left(\frac{kq_1}{r_{13}}\right) + \left(\frac{kq_2}{r_{23}}\right) - 0\right]$$

$$W = 4\ 10^{-6}\left[\frac{(9\times10^9)(1\times10^{-6})}{2\times10^{-3}} + \frac{(9\times10^9)(-2\times10^{-6})}{1\times10^{-3}}\right]$$

$$= 18 - 72 = -54 \text{ J}$$

Much as in step two, the work to place charge 3 equals the magnitude of charge 3 times the change in electric potential. Once again, the initial electric potential is 0. The final potential is the potential due to charge 1 plus the potential due to charge 2. Be careful, because the distances must be from charge 1 to charge 3 (call this r_{13}) and from charge 2 to charge 3 (call this r_{23}), respectively.

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4) Add the work from steps 1, 2, and 3.

$$W_{net} = 0 - 18 \text{ J} - 54 \text{ J} = -72 \text{ J}$$

Add the work from steps 1, 2, and 3 to find the net work. The net work is negative, meaning that the potential energy of the system has been lowered.

Similar Questions

1) A 1μC charge sits 1 cm from a −2μC charge. How much work is done in tripling the distance between these charges?
2) How much work is done in assembling a square-shaped charge distribution with a side length of 1 μm if all of the charges have a charge of 5 nC?
3) Charges 1, 2, and 3 are lined up, in that order, at 1 mm intervals along the *y*-axis. Three charges are lined up along the *y*-axis. Charge 1 has a charge of +4 μC. Charge 2 has a charge of −2 μC. Charge 3 has a charge of −3 μC. What is the change in potential energy of the system if charge 1 is removed?