Key Concepts

Chapter 5

Pressure

Density

Bernoulli's equation:

$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant (N/m}^2)$

Continuity equation: $Av =$ constant

Takeaways

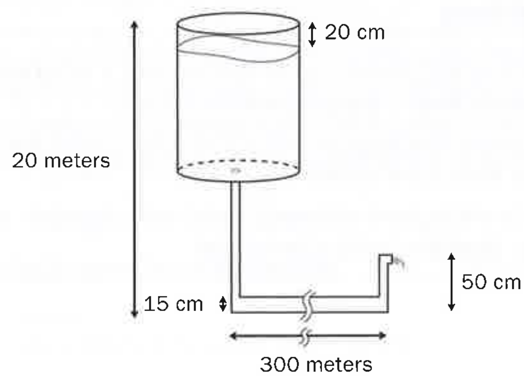
Bernoulli's equation looks complicated, but it is really just a statement of the conservation of energy. The process is the same for every problem: (1) Write Bernoulli's equation at the two points of interest, (2) eliminate any variables if possible (often via the continuity equation), and (3) solve for the unknown quantity.

Things to Watch Out For

A common use of Bernoulli's equation is with no change in height, so that $P + 1/2\rho v^2 =$ constant. In this situation, a decrease in pressure causes an increase in velocity. This is known as the Bernoulli effect and is responsible for balls curving in flight, windows exploding during hurricanes, and (partially) for airplane wings experiencing lift.

Fluid Dynamics

A water storage tank is located 300 m away from a water outlet, as shown in the diagram below. The empty space in the water tank is held at a pressure of 3 atm. The storage tank has a diameter of 5 m, and the outlet has a diameter of 1 cm. What is the speed of the water exiting the outlet? (1 atm = 101 kPa; $\rho_{water} = 1{,}000\ \text{kg/m}^3$)



1) Write an expression using Bernoulli's equation.

generic: $P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$

Bernoulli's equation is a statement of the conservation of energy for fluids. It has three terms: one analogous to kinetic energy, one analogous to potential energy, and one for pressure (a form of stored energy).

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

Write the expression for Bernoulli's principle at the two points of interest, just as you would write the total energy of a mechanical system at two points. For this problem, the two points are the top of the water level in the storage tank and the outlet.

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2) Use the continuity equation.

generic: $Av = \text{constant}$

$A_1 >> 0$, so $v_1 \approx 0$.

In almost all of Bernoulli's equation applications, you will need to eliminate some of the terms in order to solve the problem. A common one here is velocity. The continuity equation relates fluid-flow velocity to area. It states that the product of the area and velocity is a constant. Because the storage tank has a very large area, we can approximate the velocity as zero. Think about it like this: The level of the water tank is moving down very slowly. This simplifies the equation.

$$P_1 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

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3) Plug in the given information and solve.

The pressure inside the tank is 3 atm. Convert this to pascals, the SI unit for pressure.

$$P_1 = 3\ (1\text{ atm}) = 3\ (101\text{ kPa}) = 303{,}000\text{ Pa}$$
$$P_2 = 1\text{ atm} = 101{,}000\text{ Pa}$$

The pressure at the outlet is 1 atm because the outlet is exposed to outside air and there is always 1 atm of pressure outside.

$$h_1 = 20 - 0.2 = 19.8\text{ m}$$
$$h_2 = 50\text{ cm} = 0.5\text{ m}$$
$$\rho = 1{,}000\text{ kg/m}^3$$
$$P_1 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

Plug in the pressures, heights, and density and solve for v_2.

$$v_2^2 = \frac{2(P_1 + \rho g h_1 - P_2 - \rho g h_2)}{\rho}$$

$$v_2^2 = \frac{2\times[303{,}000 + (1{,}000)(9.8)(19.8) - 101{,}000 - (1{,}000)(9.8)(0.5)]}{1{,}000}$$

$$v_2^2 = 782.3$$
$$v_2 = 28.0\text{ m/s}$$

Remember: *Any pipe that is exposed to the outside will have a pressure of 1 atm. When using Bernoulli's equation, though, remember to convert this to pascals! Don't get bogged down with arithmetic: Estimate whenever possible.*

Similar Questions

1) The pressure at one point in a horizontal pipe is triple the pressure at another point. How do the fluid velocities compare at these two points?
2) Pipe A has twice the radius of pipe B. Both pipes are placed horizontally and are subjected to a fluid pressure of 1.6 atm. What is the ratio of fluid velocities in these two pipes?
3) A water storage tank is open to air on the top and has a height of 1 meter. If the tank is completely full, and a hole is made at the center of the wall of the tank, how fast will the water exit the tank?