Key Concepts

Chapter 5

Pascal's principle

Work: $W = Fd$ (N·m)

Gravitational potential energy: $U = mgh$ (N·m)

$P = \frac{F}{A}$ (N/m²)

Takeaways

Every time you see hydraulics problems, get ready to set up ratios involving the areas of the cylinders in order to calculate forces, distances, or volumes. All of the same rules of work and energy apply to hydraulics, and 100 percent efficiency is assumed. Remember: The less force used, the greater the distance.

Things to Watch Out For

There are multiple ways to solve hydraulics problems. To avoid confusion, pick a plan at the beginning and stick with it.

Hydraulic Lift

An automobile hydraulic lift consists of two circular pistons, one with a radius of 25 cm and the other with a radius of 75 cm. They are connected via a tube filled with an incompressible fluid. A constant force is applied to the smaller piston in order to raise a car with a mass of 2,000 kg to a height of 0.5 m. What is the minimum force applied to the smaller piston? How far is the smaller piston compressed?

1) Find the work performed.

The amount of work performed is simply the change in potential energy of the automobile, which equals the potential energy of the automobile at 0.5 meters. It is common to think that there is a more complex relationship because the hydraulic lift is being used, but the idea is that the hydraulic lift has 100 percent efficiency and thus all of the work put into it is used to raise the car.

$$W = mgh = (2{,}000)(9.8)(0.5) = 9{,}800 \text{ J}$$

2) Find the force on the larger piston.

The work performed is equal to the force times the distance. You can solve for the force from this formula. However, it is simpler to realize that the minimum force is the weight of the car, which equals *mg*. Realizing this will save you a step of calculation!

$$W = Fd$$
$$9{,}800 \text{ J} = F(0.5)$$
$$F = 19{,}600 \text{ N}$$

3) Find the area of the pistons.

A hydraulic lift is useful because the force is multiplied by the ratio of the areas of the pistons. Any time you see a hydraulic lift problem, count on needing to calculate the areas. You will see in step 4 that explicit calculation of the area is generally not needed, even though it is performed here.

$$A_1 = \pi r_1^2 = \pi(0.25)^2 = 0.196 \text{ m}^2$$
$$A_2 = \pi r_2^2 = \pi(0.75)^2 = 1.77 \text{ m}^2$$

4) Set the pressure in the pistons equal to each other and solve.

$$P_1 = \frac{F_1}{A_1} = P_2 = \frac{F_2}{A_2}$$

Pascal's principle states that the pressure on both pistons must be equal. Set them equal and solve, using $P = \frac{F}{A}$, the general formula for pressure.

$$F_1 = \left(\frac{A_1}{A_2}\right) F_2 = \frac{\pi(0.25)^2}{\pi(0.75)^2} \times 19{,}600 = \left(\frac{0.25}{0.75}\right)^2 \times 19{,}600$$

$$F_1 = \left(\frac{1}{3}\right)^2 \times 19{,}600 = \frac{1}{9} \times 19{,}600 = 2{,}178 \text{ N}$$

Note that the calculation can be simplified somewhat by leaving the expressions for area in terms of π. This type of thinking will save you time on Test Day.

---

5) Calculate the compression of the small piston.

Fluid is pushed through the tube; none is allowed to escape, nor is it compressed. This means that the volume of fluid moved by each piston must be the same. Set them equal to each other and solve.

$$V_1 = V_2$$

$$d_1 A_1 = d_2 A_2$$

$$d_1 = \left(\frac{A_2}{A_1}\right) d_2 = (9) \times 0.5 = 4.5 \text{ m}$$

This highlights a drawback of hydraulic lifts—even though the force is greatly reduced, the distance is increased. This is because the amount of work performed, given by $W = Fd$, is the same for both pistons:

$$W_1 = F_1 d_1 = (2{,}178)(4.5) = 9{,}800 \text{ J}$$

(Thus, another way to calculate distance is to set the work done by one piston equal to the work by the other piston.)

Similar Questions

1) A hydraulic lift has a cylinder with a radius of 5 cm. What should be the radius of the other cylinder so that the force applied to the first cylinder is multiplied by a factor of 10?
2) The small piston in a hydraulic lift with a piston radius ratio of 10:1 is compressed by 50 cm. How far does the large piston move?
3) What is the volume of fluid moved when the large piston of a hydraulic press is moved 0.2 m (the radii of the pistons are 20 cm and 80 cm)?