Key Concepts

Chapter 3

Kinetic energy

Potential energy

Momentum

Collision

$U = mgh$ (N · m: kg · m²/s²)

$K = \frac{1}{2}mv^2$ (N · m: kg · m²/s²)

$\mathbf{p} = m\mathbf{v}$ (kg · m/s)

Conservation of energy

Conservation of momentum

Takeaways

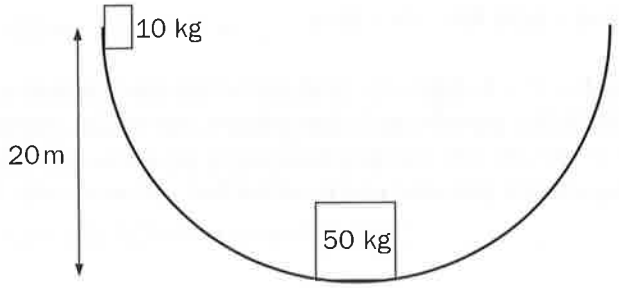
This problem combines a conservation of energy problem with a conservation of momentum problem. Remember that the conservation of energy is useful for finding velocities or distances, whereas momentum is used exclusively to find the conditions after a collision.

This is an inelastic collision. Momentum is conserved, while energy is not. Therefore, the maximum height reached should be significantly less than the initial height of the block.

Collisions and Energy

A 10 kg block starts from rest at a height of 20 meters and slides down a frictionless, semicircular track. The block collides with a stationary object of 50 kg at the bottom of the track. If the objects stick together on collision, what is the maximum height that the block-object system could reach?

1) Draw a rough sketch of the collision.



Once you draw the semicircle, draw two objects: one along the top of the semicircle and one at the bottom of the semicircle. Both objects start at rest. The 10 kg block will travel down and reach a velocity **v** before impact with the object. The objects will stick together and travel up the semicircle to a maximum height h. The question asks you to find h.

2) Write an expression for the initial energy of the falling block.

$E_i = U_i + K_i$

$U = mgh$

$K = \frac{1}{2}m\mathbf{v}^2$

$E_i = mgh_i + \frac{1}{2}m\mathbf{v}_i^2$

$E_i = mgh_i$

Write an expression for the energy of the falling block. The total energy of the block is its kinetic energy plus its potential energy. The block is initially at rest, so it has no kinetic energy.

3) Write an expression for the final energy of the falling block.

$$E_f = U_f + K_f$$
$$E_f = 0 + \frac{1}{2}m\mathbf{v}_f^2$$

Write an expression for the energy of the falling block just before it collides with the other block. At this point it has no potential energy; it has only kinetic energy.

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4) Set the expressions equal and solve for velocity.

Due to the conservation of energy, we can set the energy at any two points equal and solve. Use the energy found in steps 1 and 2 to solve for the velocity of the block just before impact.

$$E_f = E_i$$
$$mgh_i = \frac{1}{2}m\mathbf{v}_f^2$$
$$\mathbf{v}_f = (2gh_i)^{\frac{1}{2}} = 19.8 \text{ m/s}$$

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5) Write an expression for the momentum of the system before the collision.

$$\mathbf{p}_{before} = m_1\mathbf{v}_1$$

Before the collision, only mass 1 is moving. The momentum of the system is entirely due to mass 1.

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6) Write an expression for the momentum of the system after the collision.

$$\mathbf{p}_{after} = (m_1 + m_2)\mathbf{v}_2$$

After the collision, both masses are stuck together and move with the same velocity.

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7) Set the expressions equal and solve for velocity.

Due to the conservation of momentum, we can set the momentum of the system before the collision equal to that after the collision. Solve for the velocity.

$$\mathbf{p}_{after} = \mathbf{p}_{before}$$
$$m_1\mathbf{v}_1 = (m_1 + m_2)\mathbf{v}_2$$
$$\mathbf{v}_2 = \frac{m_1\mathbf{v}_1}{(m_1+m_2)} = \frac{10(19.8)}{(10+50)} = 3.3 \text{ m/s}$$

Remember: *Momentum is conserved in all types of collisions: elastic, inelastic, or completely inelastic. Energy is only conserved in elastic collisions (perfect bouncing). Because this is not an elastic collision, you cannot use energy to calculate the velocity after impact.*

Things to Watch Out For

Momentum is conserved in all types of collisions in all cases! Energy is conserved only in elastic collisions.

Similar Questions

1) In the above question, determine the height reached by each object if the collision were inelastic and the falling mass rebounded back with a speed of 1 m/s.
2) A man of mass 140 kg standing on a frictionless surface throws a 10 kg rock horizontally away from himself. What is the momentum of the system immediately after the throw?
3) Two baseballs undergo a head-on collision. Ball 1 is twice as heavy as ball 2. Ball 1 was traveling at an initial speed of $\mathbf{v}_1$, while ball 2 had an initial speed of $\mathbf{v}_2$. The type of collision was elastic. If ball 1 travels at a speed of $\frac{7}{5}\mathbf{v}_1$ after impact, what is the speed of ball 2?

8) Write an expression for the energy of the system just after the collision.

$$E_a = U_a + K_a$$

$$E_a = 0 + \frac{1}{2}(m_1 + m_2)\mathbf{v}_2^2$$

The energy of the system after the collision is due to the kinetic energy of the two blocks moving together.

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9) Write an expression for the energy of the system at the top.

$$E_t = U_t + K_t$$

$$E_t = (m_1 + m_2)gh + 0$$

When the blocks get to the top, they stop moving briefly (before falling back down), so their kinetic energy is zero.

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10) Set the expressions equal and solve for height.

$$E_a = E_t$$

$$\frac{1}{2}(m_1 + m_2)\mathbf{v}_2^2 = (m_1 + m_2)gh$$

$$\frac{1}{2}\mathbf{v}_2^2 = gh$$

$$h = \frac{v_2^2}{2g} = \frac{3.3^2}{2(9.8)} = 0.56\,\text{m}$$
