Key Concepts

Chapter 3

Gravitational potential energy:
$U = mgh$ (N·m: kg·m²/s²)

Kinetic energy:
$K = \frac{1}{2}mv^2$ (kg·m²/s²)

Collisions

Conservation of mechanical energy: $E_i = E_f$

Takeaways

Gravity is a conservative force, and at any point, the total energy is found by adding the gravitational and potential energies. In an elastic collision, kinetic energy (as well as momentum) is conserved.

Elastic Collisions

A circus performer weighing 700 N steps off a platform 9 m high. She lands on a seesaw to launch her 630 N partner straight into the air. Compare the landing and launching velocities. Find the height her partner achieves. Ignore the height of the seesaw and any dissipative forces.

1) Find the potential energy of the first performer.

$U = mgh$

$U = (700\text{ N})(9\text{ m}) = 6{,}300\text{ N}\cdot\text{m}$

Gravitational potential energy is given by the equation $U = mgh$. Because weight is the product of mass and gravitational acceleration, we need only multiply the performer's weight by her height.

Remember: *A joule is defined as a newton-meter.*

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2) Find the landing velocity of the first performer.

$E = U + K$

$K = \frac{1}{2}mv^2$

$E = 6{,}300\text{ N}\cdot\text{m}$

$6{,}300\text{ N}\cdot\text{m} = \left(\frac{1}{2}\right)(70\text{ kg})\mathbf{v}^2 \rightarrow \mathbf{v} \approx 13.4\text{ m/s}$

Because gravity is a conservative force, the total energy is the sum of the potential and kinetic energies. When the performer is atop the platform, all her energy is potential. Thus, $E = 6{,}300$ N·m. Just before the performer hits the seesaw, all of her energy is kinetic. The equation $K = \frac{1}{2}mv^2$ is used to find her landing velocity.

Remember: *On the MCAT, you will use 10 m/s² for the acceleration due to gravity, as in the solution to this question.*

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3) Find the launching velocity of the second performer.

$6{,}300\text{ N}\cdot\text{m} = \left(\frac{1}{2}\right)(63\text{ kg})\mathbf{v}^2 \rightarrow \mathbf{v} \approx 14.1\text{ m/s}$

All of the first performer's kinetic energy is transferred to the second performer. Thus, she also has 6,300 J. She has a smaller mass, so it makes sense that her velocity is higher.

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4) Find the maximal height of the second performer.
After being launched into the air, the second performer's kinetic energy is transferred back to potential energy. Compare the total energy of the system with her weight to find the maximal height she reaches.

$$6{,}300 \text{ N·m} = (630 \text{ N})h \rightarrow h = 10 \text{ m}$$

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5) Here is an alternate solution.

$U_i = U_f$
$U = mgh = Wh$
$(700 \text{ N})(9 \text{ m}) = (630 \text{ N})h \rightarrow h = 10 \text{ m}$

At any point, $E = U + K$. When the first performer hits the seesaw, all of her energy is kinetic. It is transferred to her partner, and as the second performer goes higher in the air, her energy increasingly becomes potential. Thus, we can simply equate the potential energy of the first performer with the second.

Things to Watch Out For

Dissipative forces, such as air resistance, heat, and sound, reduce the total mechanical energy of the system such that $E_f < E_i$.

Similar Questions

1) The first step in the fusion reaction that occurs on the sun is $^1H + {}^1H \rightarrow {}^2H +$ antielectron + neutrino. This step is rarely ever successful. If an unsuccessful collision of hydrogen nuclei is considered to be elastic, and they each have a mass of 1.008 amu, how do their kinetic energies compare before and after the collision?
2) A frictionless, vertical wire has two metal beads on it. The beads are held 30 cm apart by horizontal magnets. If the top magnet is removed, the first bead falls under the force of gravity and strikes the second. The first bead bounces back up to a height of 10 cm, and the second is knocked free of the magnet and falls downward. If the two beads each have a mass of 49.7 g, what is the kinetic energy of the second immediately after impact?
3) Two adult bighorn rams butt heads in an elastic collision. The alpha male (136 kg) moves more slowly than his challenger (113 kg). If the challenger collides at 8 m/s and is repelled at 6 m/s, find the kinetic energy of the system and the percent increase in speed that the alpha male experiences.