Key Concepts

Chapter 3

Momentum: $\mathbf{p} = m\mathbf{v}$

Conservation of momentum: $\Sigma p_i = \Sigma p_f$

Collisions

Vector addition

$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\mathbf{v}_f$

Takeaways

Conservation of momentum will get you through any collision problem on the MCAT. Remember to break motion down into **X** and **Y** components if an angle is involved.

Conservation of Momentum

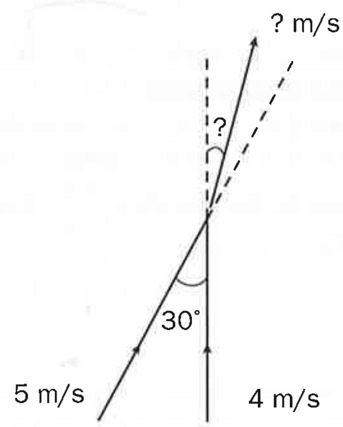
A rugby player with a mass of 80 kg is running due north with a speed of 4 m/s. He is hit by a 90 kg rival at 5 m/s at an angle of 30° from the south. The two players move together with an unknown velocity before falling to the ground. Find their combined speed and direction.

1) Determine the type of collision.

The question stem states that the two players move together after impact. This indicates an inelastic collision. Energy is lost in an inelastic collision as heat, sound, deformation, and so on, so we cannot use the equation for conservation of kinetic energy. We can, however, use the equation for conservation of momentum.

---

2) Draw vectors representing the collision.

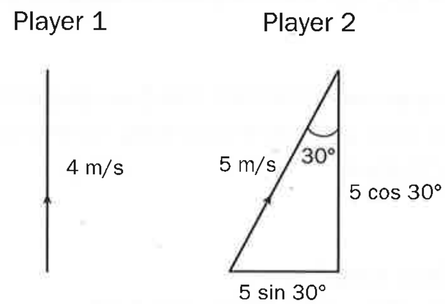


Because we are dealing with angles, we'll need to break the velocity vector down into **X** and **Y** components. First we need to sketch what those components will be.

Remember: *Think critically. If the second player hits the first at a 30° angle, the final angle should be between 0 and 30° from the north. Even if the second player had a significantly greater momentum, the final angle could not be greater than the initial one!*

---

3) Break the vectors into X and Y components.



Player 1 is moving due north, so all 4 m/s of his speed is oriented upward. Player 2, however, is moving at an angle. We must consider how much of his momentum moves right and how much moves up. Break his velocity into **X** and **Y** components.

Remember: *The mnemonic SOH CAH TOA will help you remember which trigonometric function to use for each component.*

---

4) Apply the equation for conservation of momentum.

$\mathbf{p}_{\text{before}} = \mathbf{p}_{\text{after}}$

$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\mathbf{v}_f \quad y\text{: } 80(4) + 90(5\cos 30°) = (80 + 90)(\mathbf{v}_f \cos\theta) \rightarrow$

$\mathbf{v}_f \cos\theta = \dfrac{320 + 389.7}{170} \approx 4.17$

$x\text{: } 80(0) + 90(5\sin 30°) = (80 + 90)(\mathbf{v}_f \sin\theta) \rightarrow \mathbf{v}_f \sin\theta = \dfrac{225}{170} \approx 1.32$

In both elastic and inelastic collisions, momentum is conserved. This means that we can set the momentum of the system before the collision equal to the momentum of the system after the collision. In this case, we are dealing with a totally inelastic collision. This means that the two masses stick together after impact and move off as a unit. Thus, the momentum of the system before the collision is $m_1\mathbf{v}_1 + m_2\mathbf{v}_2$, and afterwards it is $(m_1 + m_2)\mathbf{v}_f$. Because this problem deals with two dimensions, we must break the velocity vectors down into **X** and **Y** components and then apply the equation for conservation of momentum to each. We end up having two equations with two variables, v_f and θ, where θ is the angle from north.

---

5) Use the relationship between sinusoidal functions to solve for θ.

$\dfrac{\mathbf{v}_f \sin\theta}{\mathbf{v}_f \cos\theta} = \dfrac{1.32}{4.17}$

$\dfrac{\sin\theta}{\cos\theta} = \dfrac{1.32}{4.17} = \tan\theta$

$\tan^{-1}(0.316) = 17.6°$

Things to Watch Out For

If you are given a situation in which two objects collide with two angles, it may prove far easier to reorient the axes of the situation so that you only have to deal with one set of vector components.

Similar Questions

1) A 1,980 kg car moving at 13 m/s is brought to a stop in 2 seconds when it collides with a wall. If a new model of this car has a longer crumple zone, the passengers experience a 3,217.5 N force upon impact. By what percentage has the period of impact been increased? Has the impulse on the car and its passengers changed?
2) A curler slides a stone across an ice rink with an initial speed of 3 m/s towards the center of a target. It strikes a second stone, which then hits a third stone. All stones have a mass of 44 kg and are hit head on. If the second and third stones move with individual final velocities of 1 m/s, find the velocity of the first stone after it collides with the second.
3) A 4.2 g bullet is fired into a stationary 5 kg block of wood. If the bullet lodges in the block, knocking it back with a speed of 0.81 m/s, find the speed of the bullet prior to impact.

To find θ, we need to get rid of $\mathbf{v}_f$ temporarily. Divide the x-equation by the y-equation. $\mathbf{v}_f$ cancels out, and the components allow us to find θ with the arc tangent.

Remember: *Use the mnemonic SOH CAH TOA if you forget how sine and cosine are related to tangent. If you divide sine by cosine, you end up with (O/H)/(A/H) = O/A, the definition of tangent.*

---

6) Use θ to find the final speed.

y: $80(4) + 90(5 \cos 30°) = (80 + 90)(\mathbf{v}_f \cos 17.6°) \rightarrow$
$\mathbf{v}_f = 4.17 \div (\cos 17.6°) \approx 4.37$ m/s
x: $80(0) + 90(5 \sin 30°) = (80 + 90)(\mathbf{v}_f \sin 17.6°) \rightarrow$
$\mathbf{v}_f = 1.32 \div (\sin 17.6°) \approx 4.37$ m/s

Plug 17.6° back into either the x- or y-equation. In both cases, we find $\mathbf{v}_f$ = 4.37 m/s. If we find different values with these equations, we should look very carefully for our mistake.

---

7) Here is an alternate solution.

$$\mathbf{p}_x = \mathbf{p}_{1x} + \mathbf{p}_{2x} = m\mathbf{v}_{1x} + m\mathbf{v}_{2x} = 0 + (90)5 \sin 30° = 225$$
$$\mathbf{p}_y = \mathbf{p}_{1y} + \mathbf{p}_{2y} = m\mathbf{v}_{1y} + m\mathbf{v}_{2y} = (80)4 + (90)5 \cos 30° = 709.7$$
$$\mathbf{p} = (\mathbf{p}_x^{\ 2} + \mathbf{p}_y^{\ 2})^{\frac{1}{2}} = (225^2 + 709.7^2)^{\frac{1}{2}} = 744.5$$
$$\mathbf{p} = (m_1 + m_2)\mathbf{v}_f \rightarrow \mathbf{v}_f = \frac{\mathbf{P}}{m_1 + m_2} = \frac{744.5}{90 + 80} = 4.37\text{m/s}$$
$$\alpha = \tan^{-1}\left(\frac{\mathbf{p}_y}{\mathbf{p}_x}\right) = \tan^{-1}\left(\frac{709.7}{225}\right) = 72.4° \rightarrow \theta = 90° - 72.4° = 17.6°$$

where α is the angle from east and θ is the angle from north.

An alternate solution is to calculate the **X** and **Y** components of the momentum directly by adding the **X** and **Y** components of the momentum of the system before the collision. Then, use the Pythagorean theorem to calculate the magnitude of the vector. Because momentum is conserved, set this equal to the momentum of the system after the collision and solve for $\mathbf{v}_f$. To find the angle, use trigonometry, and note that to find the angle relative to the horizontal, you must find the complementary angle.
