## Key Concepts

Chapter 3

Conservation of energy

Centripetal acceleration

Gravitational potential energy: $U = mgh$ (J)

Kinetic energy: $K = \left(\frac{1}{2}\right)mv^2$ (J)

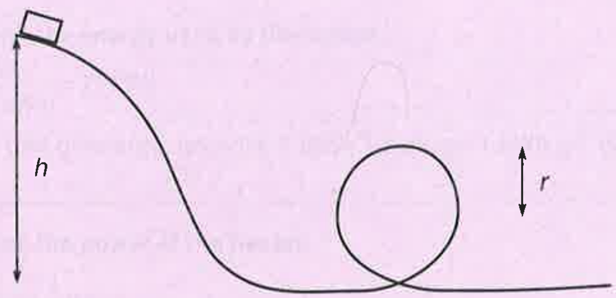
## Takeaways

The key to this problem is knowing that the normal force is zero at the top of the loop in the case where the block is just about to fall.

Notice that the loop problem follows the same process as any other conservation of energy problem but with the added aspect of centripetal acceleration. Draw a free-body diagram and solve for the velocity. After this, your goal is the same as always: (1) write expressions for the energy at two points and (2) set them equal and solve for the unknown quantity.

# Circular Loops

A 1 kg block slides down a ramp and then around a circular loop of radius 10 m, as shown in the diagram below. Assuming that all surfaces are frictionless, what is the minimum height of the ramp required so that the block makes it all the way around the loop without falling?




**1) Write an expression for the initial energy of the system.**

At the top of the ramp, the block has only potential energy given by the formula $U = mgh$.

$$E_i = mgh$$

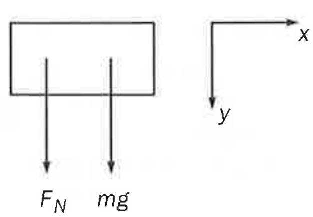
***Remember:*** *Leaving the expressions in terms of variables will save time and reduce the chance of calculation error.*

**2) Write an expression for the energy of the system at the top of the loop.**

At the top of the loop, the block has both potential energy and kinetic energy. The height of the block at the top of the loop is $2r$. Add these to get the total energy.

$$E_{loop} = mg(2r) + \frac{1}{2}mv^2$$

**3) Draw a free-body diagram of the block at the top of the loop.**




There are two forces acting on the block at the top of the loop: the weight of the block (equal to $mg$) and the normal force (labeled $F_N$). Note that the normal force is acting down because the track is above the mass.

***Remember:*** *The normal force is always perpendicular to the surface and points from the surface to the object.*

**4) Add the forces in the $y$ direction.**
There are no forces acting in the $x$-direction, so add the forces in the $y$-direction only.

$$\Sigma F_y = ma_y = mg + F_N$$

**5) Set the normal force equal to zero.**
If the block falls off of the ramp at the top of the loop, the normal force will become zero because the ramp is no longer touching the block. By setting the normal force equal to zero, we are solving for the case where the block just starts to fall.

$$F_N = 0 \rightarrow ma_y = mg$$

**6) Identify the acceleration as centripetal.**
Because the block is traveling in a circle, it has an acceleration directed towards the center of the circle, which is called centripetal acceleration. The magnitude of this acceleration is given by the formula $a_c = \frac{v^2}{r}$. Plug this into the equation from step 5 and solve for $v$.

$$a_y = \frac{v^2}{r} \rightarrow \frac{mv^2}{r} = mg$$
$$v = (gr)^{\frac{1}{2}}$$

**7) Set the energy expressions equal and solve.**
Due to the conservation of energy, we can set the energy at any two points as equal. Do this and set the velocity at the top of the loop equal to the value from step 6. Then solve for $h$. Because we have set the velocity equal to that at which the block starts to fall off the ramp, we have solved for the minimum height of the ramp.

$$E_i = E_{\text{loop}}$$
$$mgh = mg(2r) + \frac{1}{2}mv^2$$
$$mgh = mg(2r) + \frac{1}{2}m((gr)^{\frac{1}{2}})^2$$
$$mgh = mg(2r) + \frac{1}{2}m(gr)$$
$$mgh = 2.5mgr$$
$$h = 2.5r = 25 \text{ m}$$

## Things to Watch Out For

Other variations to this problem include solving for the normal force at various points on the loop, adding friction to the ramp, or having multiple changes in elevation. They are all solved using the same process.

## Similar Questions

1) What is the normal force at the bottom of the loop if the height of the ramp is four times that of the radius of the loop?
2) How fast does the block need to be going at the bottom of the ramp so that the acceleration of the block at the top of the loop is $4g$?
3) What is the speed of the block as it exits the loop if the normal force at the top of the loop was 80 N?