Key Concepts

Chapter 2

Centripetal acceleration

Friction

Newton's laws

$f \leq \mu_s F_N$

$a_c = \frac{v^2}{r}$ (m/s²)

Takeaways

Do not be scared off by the circular motion of this problem. It only adds one step to the solution! The process for solving this problem is the same as for any force problem: Draw the forces on the object; write the sum of forces in the *x*- and *y*-directions; set these equal to ma_x and ma_y, respectively; and then solve. The only difference with this problem is that you set the acceleration equal to $\frac{v^2}{r}$ at the end.

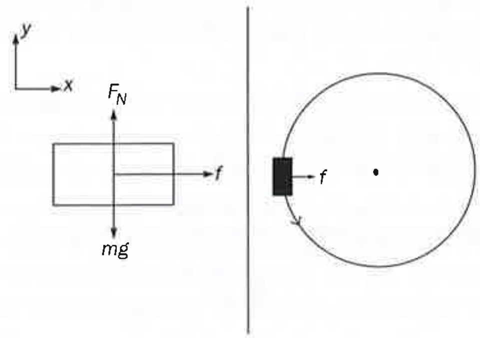
Things to Watch Out For

A more complicated version of this problem would have the car traveling around an inclined track. Then we would have to break the weight of the car into its components, much as in an inclined-plane problem.

Uniform Circular Motion

A car rounds a curve with a constant velocity of 25 m/s. The curve is a circle of radius 40 m. What must the coefficient of static friction between the road and the wheels be to keep the car from slipping?

1) Draw a free-body diagram of the car.



There are three forces acting on the car: the weight of the car, which equals *mg* (labeled *mg*); the normal force (labeled F_N); and the force of friction (labeled *f*). The view shown on the left is a head-on view of the car. The force of friction points toward the center of the circle, as shown in the diagram on the right.

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2) Add up the forces in the *x*- and *y*-directions.

$$\Sigma F_x = ma_x = f$$
$$\Sigma F_y = ma_y = F_N - mg$$

The sum of the forces in a given direction is always equal to *ma*. This is Newton's second law.

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3) Solve for the normal force.

We know that the car is not accelerating the in *y*-direction because it is not sinking into the road or coming off of the road; thus, we can set $a_y = 0$ and solve for F_n.

$$a_y = 0\text{, so } \Sigma F_y = 0\text{: } F_N - mg = 0$$
$$F_N = mg$$

Remember: *Solving for the normal force is generally only necessary when the problem involves friction.*

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4) Write the force of friction in terms of the normal force.

The maximum force of static friction is the coefficient of friction, μ_s, multiplied by the normal force. Plug the expression for normal force from step 3 into the force equation from step 2.

generic: $f \le \mu_s F_N$

$f = ma_x$

$\mu_s F_N \ge ma_x$

$\mu_s mg \ge ma_x$

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5) Identify a_x as the centripetal acceleration and solve.

Whenever anything travels in a circle, it has an acceleration directed toward the center of that circle, called the centripetal acceleration. This acceleration has a magnitude given by $a_c = \frac{v^2}{r}$.

generic: $a_c = \frac{v^2}{r}$

In this problem, the car is going in a circle, and the center of the circle is always pointed in the x-direction. This means that the car is accelerating in the x-direction, and we can set $a_x = \frac{v^2}{r}$. Substitute $\frac{v^2}{r}$ for a_x into the final equation from step 4, and solve for μ_s.

$a_x = \frac{v^2}{r}$, so $\mu_s mg \ge \frac{mv^2}{r}$

$\mu_s g \ge \frac{v^2}{r}$

$\mu_s \ge \frac{v^2}{gr} = \frac{(25)^2}{(9.8 \times 40)} = 1.59$

Similar Questions

1) What is the minimum radius that a cyclist can ride around at 10 kilometers per hour without slipping if the coefficient of friction between her tires and the road is 0.5?
2) A 1 kg ball is swung around in a 90 cm circle at an angle of 10° below the horizontal. What is the tension in the string?
3) A force how many times greater than that of gravity is felt by a race car driver rounding a turn with a radius of 50 m at a speed of 120 m/s?