Key Concepts

Chapter 2

Pulley

Inclined plane

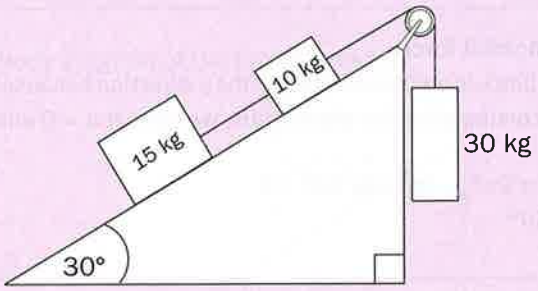
Tension

Takeaways

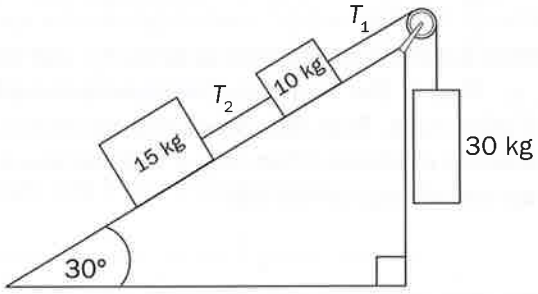
This was a complicated problem solved methodically. On Test Day, be sure to draw the force diagram for each object. Do not be intimidated by multiple objects on a pulley system. It does not matter which way you assume the acceleration of the objects to go. If you chose incorrectly, the acceleration you get will be negative. Just remember to reverse the direction.

Pulley with Multiple Masses

What is the tension on the rope between the 10.0 kg and 15.0 kg masses? (Assume no friction.)



1) Assign tensions of the ropes.



Set down as positive. Tension is a pull.

Let T_1 be the tension in the rope connecting the 10.0 kg and 30.0 kg masses.
Let T_2 be the tension in the rope connecting the 10.0 kg and 15.0 kg masses.

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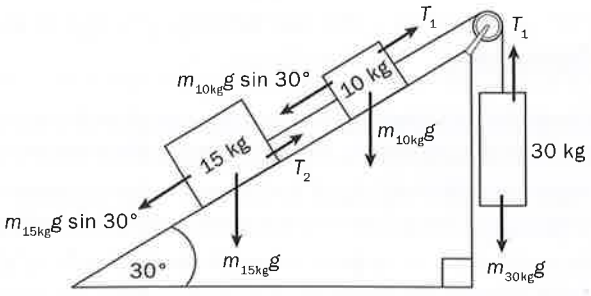
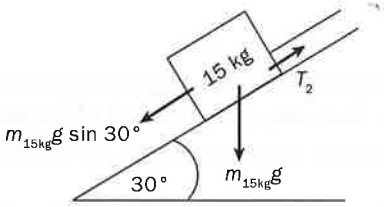
2) Draw force diagrams on each block system.



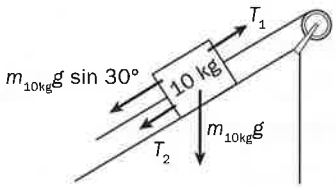
Establish down as positive. Because the right side of the pulley system is heavier than the left side, let us assume that the right object falls down while the other two objects accelerate towards the pulley. Thus, the two objects on the left have negative accelerations, whereas the object on the right has a positive acceleration.

Add the sum of the forces on each object to get a net force equal to mass times acceleration.

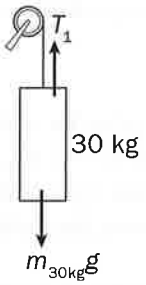
Sum of the forces:



15 kg object: $F_{net} = m_{15kg}g\sin 30° - T_2 = -m_{15kg}a$



10 kg object: $F_{net} = m_{10kg}g\sin 30° - T_1 + T_2 = -m_{10kg}a$



30 kg object: $F_{net} = m_{30kg}g - T_1 = m_{30kg}a$

Remember: *It does not matter which way you assign your objects to move, as long as one side is moving up while the other side is moving down. Whatever sign you assign acceleration for one side, the other side needs to be opposite in sign. When you get an answer, compare it to the signs you've assigned.*

Similar Questions

1) Suppose there was a coefficient of friction of 0.1 between the blocks and the inclined plane for the same problem above. Solve for the tension between the 10 kg mass and the 15 kg mass.
2) What is the tension on the rope between the 10 kg object and the 30 kg object?
3) Suppose that the incline on the plane is now 90°. What is the magnitude and direction of acceleration of the 30 kg object?

3) Solve the equations simultaneously.
(30 kg object solved for T_1)

$$T_1 = m_{30kg}g - m_{30kg}a$$
$$m_{10kg}g\sin 30° + T_2 - m_{30kg}g + m_{30kg}a = -m_{10kg}a$$

(Previous equation solved for T_2)

$$T_2 = m_{30kg}g - m_{10kg}a - m_{30kg}a - m_{10kg}g\sin 30°$$
$$m_{30kg}g\sin 30° - m_{30kg}g + m_{10kg}a + m_{30kg}a + m_{10kg}g\sin 30° = -m_{15kg}a$$

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4) Get all *a* terms on the left side and all *g* terms on the right side.
Take the last equation from step 3, and get all of the acceleration terms on the left-hand side while leaving the gravity terms on the right-hand side.

$$m_{10kg}a + m_{30kg}a + m_{15kg}a = m_{30kg}g - m_{10kg}g\sin 30° - m_{15kg}g\sin 30°$$

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5) Solve for the tension of the rope.

$$a(m_{10kg} + m_{30kg} + m_{15kg}) = m_{30kg}g - m_{10kg}g\sin 30° - m_{15kg}g\sin 30°$$
$$a = \frac{(m_{30kg}g - m_{10kg}g\sin 30° - m_{15kg}g\sin 30°)}{(m_{10kg} + m_{30kg} + m_{15kg})}$$

$$a = \frac{(30.0\text{ kg} \times 9.80\text{ m/s}^2 - 10.0\text{ kg} \times 9.80\text{ m/s}^2 \times \sin 30° - 15.0\text{ kg} \times 9.80\text{ m/s}^2 \times \sin 30°)}{(10.0\text{ kg} + 30.0\text{ kg} + 15.0\text{ kg})}$$

$$a = 3.12\text{ m/s}^2$$

From step 4, factor out *a*. Then solve for *a* to find the acceleration. Plugging in the numbers gives an acceleration of 3.12 m/s^2. This confirms our earlier establishment that the object to the right of the pulley is going down. Because we declared that *a* was positive down, the right object will be accelerating down.

Once you have determined *a*, use the 15 kg object equation to solve for T_2.

15 kg object equation:

$$m_{15kg}g\sin 30° - T_2 = -m_{15kg}a$$
$$T_2 = m_{15kg}a + m_{15kg}g\sin 30°$$
$$T_2 = 15.0\text{ kg} \times 3.12\text{ m/s}^2 + 15.0\text{ kg} \times 9.80\text{ m/s}^2 \times \sin 30°$$
$$T_2 = 120\text{ N}$$
