Key Concepts

Chapter 2

Newton's laws

Friction

Kinematics

Newtonian mechanics

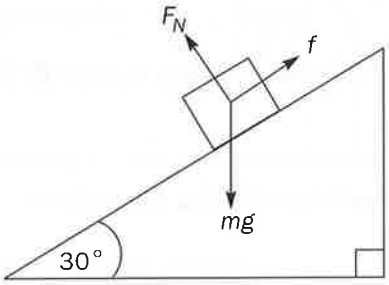
$W_x = mg \sin \theta$, $W_y = mg \cos \theta$

$\Delta x = v_0 t + \frac{1}{2} a t^2$

Inclined Plane

A block with a mass of 2 kg is sliding down a plane that is inclined at 30° to the horizontal. The coefficient of kinetic friction between the block and the plane is 0.3. Starting from rest, how far does the block travel in 2 seconds?

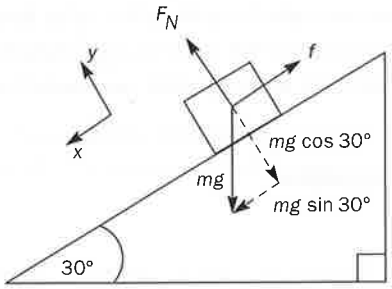
1) Draw a free-body diagram of the forces present.



There are three forces acting on the block: the force of gravity (which equals *mg*), which always acts straight down; friction (labeled *f*), which always acts opposite the direction of motion; and the normal force (labeled F_N), which is always perpendicular to the plane.

---

2) Break the weight into X and Y components.



X component of *mg*:	*mg* sin 30°
generic:	*mg* sin θ
Y component of *mg*:	*mg* cos 30°
generic:	*mg* cos θ

Orient the *x*–*y* axis so that the *x*-axis points down the plane. This will make the calculations much easier because we can solve for the acceleration in the *x*-direction. F_N points in the +*y*-direction, and *f* points in the –*x*-direction. The weight, *mg*, must be broken into components along the axes.

Remember: *When the angle of the plane is given with respect to the horizontal, the component of the weight along the plane will always be* mg *sin* θ.

3) Write the sum of the forces in each direction.

$\Sigma F_x = ma_x = mg \sin 30° - f$

$\Sigma F_y = ma_y = F_N - mg \cos 30°$

The sum of the forces in a given direction is always equal to *ma*. This is Newton's second law.

---

4) Solve for the normal force.

We know that the block is not accelerating in the *y*-direction because it is not sinking into the plane or coming off of the plane; thus, we can set $a_y = 0$ and solve for F_N.

$a_y = 0$, so $\Sigma F_y = 0$: $F_N - mg \cos 30° = 0$

$F_N = mg \cos 30°$

---

5) Solve for the acceleration of the block.

$f = \mu_k F_N = \mu_k mg \cos 30°$

$ma_x = mg \sin 30° - \mu_k mg \cos 30°$

$a_x = g \sin 30° - \mu_k g \cos 30° = 9.8\ (0.5) - 0.3(9.8)(0.866) = 2.35\ \text{m/s}^2$

The force of friction depends on the normal force, F_N, and the coefficient of kinetic friction, μ_k. Plug in the expression for normal force from step 4 to determine the friction force. Plug this expression for friction force into the forces in the *x*-direction expression from step 3 to determine the acceleration. Note that the mass cancels out completely.

---

6) Use a kinematics formula to calculate the distance.

$$\Delta x = v_0 t + \frac{1}{2}at^2$$

$$\Delta x = 0 + \frac{1}{2}(2.35)(2)^2 = 4.7 \text{ meters}$$

This is a general kinematics formula that you should have memorized. In this particular problem, the initial velocity of the block, v_0, is zero.

Takeaways

In every force problem, the process is the same: Draw the forces on the object; write the sum of forces in the *x*- and *y*-directions; set these equal to ma_x and ma_y, respectively; and then solve. In most single-body problems, the mass will cancel out of the equation. Notice that this problem would be much simpler if the ramp were frictionless.

Things to Watch Out For

The geometry can become confusing on Test Day, so it is helpful to memorize that the component of weight along the plane is $mg \sin \theta$, as long as θ is the angle to the horizontal.

Similar Questions

1) A block of mass 5 kg is placed on an inclined plane at 45° to the horizontal. What is the minimum coefficient of static friction so that the block remains at rest?
2) A block is given an initial velocity of 2 m/s up a frictionless plane inclined at 60° to the horizontal. What is the highest point reached by the block?
3) What is the velocity of a 10 kg block down a frictionless inclined plane at 30° to the horizontal 5 seconds after it is released from rest?