Key Concepts

Chapter 2

Newton's laws

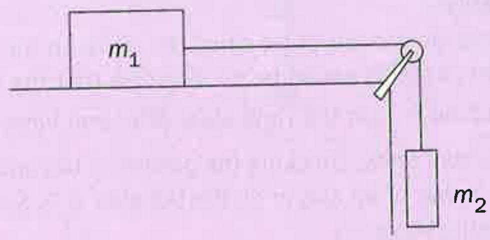
Friction

Pulley

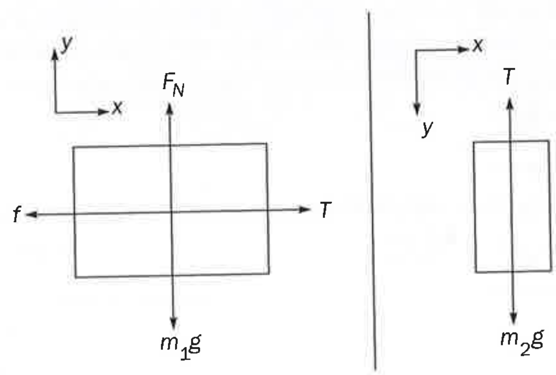
Tension

Two Connected Masses

Two masses are connected by a string over a massless pulley as shown below. The coefficient of kinetic friction between mass 1 and the table on which it sits is 0.3. If the system is released from rest, what is the acceleration of mass 1? (m_1 = 1 kg; m_2 = 4 kg.)



1) Draw free-body diagrams of both masses.



On mass 1, there are four forces acting: the normal force (labeled F_N), the weight (which equals m_1g), the tension in the string (labeled T), and the friction force (labeled f).

On mass 2, there are two forces acting: the weight (labeled m_2g) and the tension in the string (labeled T).

Notice that for mass 2, we have chosen the positive y-direction as down. Because we know that mass 2 will be going downward, this will simplify the calculations in later steps.

Takeaways

The key here is that if there are two masses involved, you need to draw two free-body diagrams and write two sets of the sum of forces. Any two-mass problem will end in solving a system of two equations.

2) Add the forces in the x- and y-direction for each mass.

Mass 1:

$\Sigma F_x = m_1 a_{x1} = T - f$

$\Sigma F_y = m_1 a_{y1} = F_N - m_1 g$

Mass 2:

$\Sigma F_x = m_2 a_{x2} = 0$

$\Sigma F_y = m_2 a_{y2} = m_2 g - T$

Things to Watch Out For

This problem could be made more complex by adding an incline or a third mass. No matter how complex the situation is, you can apply the same problem-solving process.

According to Newton's second law, the sum of the forces in a given direction is always equal to the mass times the acceleration in that direction. Note that the tension (T in the diagrams) is the same tension in both equations.

3) Solve for the normal force of mass 1.
We know the acceleration in the y-direction is zero because mass 1 cannot move in the y-direction. Set $a_{y1} = 0$ and solve.

$a_{y1} = 0$, so $\Sigma F_y = 0$: $F_N - m_1 g = 0$
$F_N = m_1 g$

Remember: *Generally, it is only necessary to solve for the normal force when friction is involved.*

4) Write the friction force in terms of the normal force.

$f = \mu_k F_N = \mu_k m_1 g$
$m_1 a_{x1} = T - \mu_k m_1 g$
$m_2 a_{y2} = m_2 g - T$

The force of friction depends on the normal force and the coefficient of friction, μ_k.

5) Relate the accelerations of mass 1 and mass 2.

$a_{x1} = a_{y2} = a$

The two masses must accelerate at the same rate because they are tied together. Because we have chosen the positive y-direction to be downward for mass 2, we can say that $a_{x1} = a_{y2}$ and simplify the notation by calling them both a. (If we had chosen the positive y-direction to be upward, we would have to say $a_{x1} = -a_{y2}$).

6) Solve the system of equations.

(1) $m_1 a = T - \mu_k m_1 g$
(2) $m_2 a = m_2 g - T$

Solve (1):

$m_1 a = T - \mu_k m_1 g \therefore T = m_1 a + \mu_k m_1 g$

Plug in to (2):

$m_2 a = m_2 g - (m_1 a + \mu_k m_1 g)$
$m_2 a + m_1 a = m_2 g - \mu_k m_1 g$
$(m_1 + m_2) a = m_2 g - \mu_k m_1 g$
$a = \dfrac{m_2 g - \mu_k m_1 g}{m_1 + m_2} = 7.25\ \text{m/s}^2$

Steps 4 and 5 leave us with two equations (labeled 1 and 2) and two unknowns (a and T). Solve for T in the first equation. Then, plug this expression for T into the second equation. Rearranging, we get an expression for acceleration. Finally, plug in the known values for m_1, m_2, g, and μ_k into the expression for a to get the final answer for the acceleration of m_1.

Similar Questions

1) Two masses are tied together by a string over a massless pulley so that they can both move vertically. Their masses are 1 kg and 3 kg. What is the acceleration of the 1 kg mass?
2) Two masses are connected by a string over a massless pulley. Mass A is on a table, while mass B hangs freely. What is the coefficient of friction necessary between mass A and the table to keep the system at rest?
3) Two masses are tied together by a string over a massless pulley so that they can both move vertically. One mass is 5 kg, and the other is 1 kg. The masses are released from rest. How far does one mass fall to reach a velocity of 10 m/s?