Key Concepts

Chapter 2

Dimensional analysis

Frequency

Density

Force

Surface tension

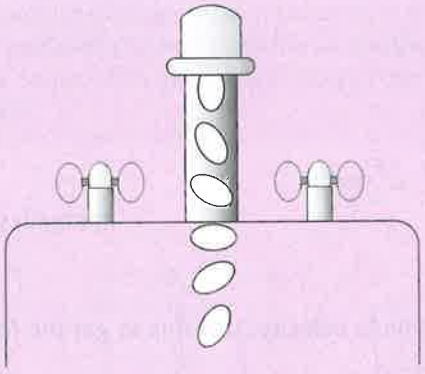
Takeaways

It is possible to find the trend for a quantity based solely on units of that quantity. Keep this in mind when facing unfamiliar topics on the MCAT.

Things to Watch Out For

If you are short on time, you can at least determine general trends using a commonsense approach. The larger the radius, the slower the drop will oscillate. The denser the fluid, the slower the drop will oscillate. The surface tension is supplying the force to keep the drop oscillating, so if it is stronger, the drop will oscillate more quickly.

Dimensional Analysis



Water is dripping from a leaky faucet. As the drops fall, they oscillate as shown in the diagram above. Given that the frequency depends only on the surface tension T of the water (measured in N/m), the radius of the drops r, and the density of the water, use dimensional analysis to find a proportionality expression for the frequency f of the drops.

1) Identify all of the relevant physical quantities and their units.

T: $N/m = kg/s^2$

ρ: kg/m^3

r: m

f: $Hz = \frac{1}{s}$

As given in the problem, surface tension has units of Newtons per meter. 1 N is 1 $kg \cdot m/s^2$. Density has units of kg/m^3. Radius is a distance with units of meters. Frequency has units of Hz, or $\frac{1}{s}$.

Remember: *If you forget what a newton is, remember* F = ma, *so newtons =* $kg(m/s^2)$.

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2) Write a hypothetical formula for the frequency.

The frequency is related to the surface tension, T, the radius, r, and the density, ρ. Write an equation for these using variables as exponents.

$f = kT^x r^y \rho^z$, where k is a unitless constant

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3) Plug the units into the hypothetical formula.
Plug the units into the hypothetical formula from step 2.

$$f = kT^x r^y \rho^z$$
$$1/s = (kg/s^2)^x (m)^y (kg/m^3)^z$$

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4) Solve for the variables.
We know that the units on the left must equal the units on the right. Only the first term on the right contains seconds, so we know that the exponent must be $\frac{1}{2}$; thus, we end up with $\frac{1}{s}$ on the right side. After you have this, set up an equation for the other two units, checking the powers of the units on each side of the equation. The power of kg and m on the left side is 0. Solve for y and z. Plug into the hypothetical formula.

$$1/s = (kg/s^2)^x (m)^y (kg/m^3)^z$$
$$x = \frac{1}{2}$$
$$\text{kg: } 0 = x + z$$
$$\text{m: } 0 = y - 3z$$
$$\Rightarrow z = -\frac{1}{2}, y = -\frac{3}{2}$$
$$\Rightarrow f = k\frac{T^{1/2}}{r^{3/2}\rho^{1/2}} = k\sqrt{\frac{T}{r^3\rho}}$$

Similar Questions

1) What are the units of G in Newton's law of gravitation?
2) An electric dipole is initially at rest in a uniform electric field. The torque provided by the electric field causes the dipole to oscillate back and forth. For the period of motion, the physicist derives the formula $T = k(E)^{\frac{1}{2}}$. In this equation, T is the period, E is the electric field, and k is a quantity with the appropriate units. Is this equation physically reasonable?
3) What are the units for the permeability of free space, μ_0?