Key Concepts

Chapter 2

Universal Law of Gravitation

Newton's laws

$F = \frac{Gm_1m_2}{r^2}$ (N = kg · m/s²)

Takeaways

The acceleration due to gravity is derived from the Universal Law of Gravitation.

Things to Watch Out For

Be familiar with the solution presented in step 4 because asking for the ratio between two quantities is a very common question type on the MCAT.

Universal Law of Gravitation

On a certain planet with radius 7,000 km, the acceleration due to gravity at the surface of the planet is 6 m/s². What is the acceleration due to gravity at an elevation above the surface equal to twice the radius of the planet? By what factor is the acceleration due to gravity changed at this elevation? ($G = 6.6 \times 10^{-11}$ N · m²/kg².)

1) Find the mass of the planet.

Use the Universal Law of Gravitation. In this equation, r is the distance from the center of the mass, so at the surface of the planet it is the radius of the planet. Use Newton's second law, $a = \frac{F}{m}$, to get an expression for acceleration. Solve for the mass of the planet.

$$F_{surf} = \frac{Gm_1m_2}{r^2}$$

$$a_{surf} = \frac{F}{m} = \frac{Gm_{planet}}{r_{planet}^2}$$

$$m_{planet} = \frac{(a_{surf})(r_{planet}^2)}{G} = \frac{(6)(7\times10^6)^2}{(6.6\times10^{-11})} = \frac{(6)(49\times10^{12})}{(6.6\times10^{-11})} = 4.5\times10^{24}\text{ kg}$$

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2) Write an expression for the acceleration at the high elevation.

$$F_{elev} = \frac{Gm_1m_2}{(r+h)^2}$$

This is the same expression as in step 1, except the distance from the center of the planet is now the radius of the planet added to the height above the surface, h. Solve for acceleration.

$$a_{elev} = \frac{F}{m} = \frac{Gm_{planet}}{(r_{planet}+h)^2}$$

$h = 2(7{,}000) = 14{,}000\text{ km} = 1.4\times10^7\text{ m}$

$$a_{elev} = \frac{(6.6\times10^{-11})(4.5\times10^{24})}{(7\times10^6+1.4\times10^7)^2}$$

$$a_{elev} = \frac{(6.6\times10^{-11})(4.5\times10^{24})}{(2.1+10^7)^2}$$

$$a_{elev} = \frac{(6.6\times10^{-11})(4.5\times10^{24})}{(4.41\times10^{24})}$$

$$a_{elev} = 0.67\text{ m/s}^2$$

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3) Divide the accelerations.

$$\frac{a_{elev}}{a_{surf}} = \frac{(0.67)}{(6)} = 0.11$$

To find the factor by which acceleration has changed, simply divide the elevated acceleration by that at the surface.

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4) Here is an alternate solution to part 2 of the question.

$$\frac{a_{elev}}{a_{surf}} = \frac{\frac{Gm_{planet}}{(r_{planet}+h)^2}}{\frac{Gm_{planet}}{r_{planet}^2}} = \frac{Gm_{planet}\, r_{planet}^2}{(r_{planet}+h)^2\, Gm_{planet}} = \frac{r_{planet}^2}{(r_{planet}+2r_{planet})^2} = \frac{r_{planet}^2}{9r_{planet}^2} = \frac{1}{9}$$

If we had not solved for the mass of the planet in step 1, we could have solved for the factor of change in acceleration by writing a fraction of the two expressions. It simplifies with algebra to $\frac{1}{9}$. It is three times the distance, and the acceleration is related to the distance squared.

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5) And here is another solution.

r is getting multiplied by 3. Using the formula $F = \frac{Gm_{planet}m_{object}}{r^2}$, we see that F is inversely proportional to r^2; therefore, F is getting multiplied by $\frac{1}{9}$. Using the formula $a = \frac{F}{m}$, we see that a is directly proportional to F; therefore, a is getting multiplied by $\frac{1}{9}$.

Similar Questions

1) Two masses, $m_1 = 10$ kg and $m_2 = 30$ kg, are 1 m apart. What is the force of gravity acting on m_1 from m_2?
2) At what height above the Earth's surface is the acceleration due to gravity 10% of that at sea level?
3) The moon has $\frac{1}{6}$ the acceleration of gravity of the Earth. What would the mass of the Earth have to be to have this acceleration at its surface (where Earth (radius) = 6378 km and Earth (mass) = 5.97×10^{24}kg)?