Key Concepts

Chapter 1

Conservation of energy

Kinetic energy

Potential energy

Nonconservative forces

Kinematics

$E_f = U_f + K_f$ (J)

Things to Watch Out For

The mention of lost energy in the question stem should tip you off that you need to use the conservation of energy equation. In general, unless you are asked for an acceleration or time value, the energy approach is easier and faster than the kinematics/Newton's laws approach.

Projectile Motion and Air Resistance

An arrow with a mass of 80 g is fired at an angle of 30° to the horizontal. It strikes a target located 5 m above the firing point and impacts the target traveling 20 m/s. If 10% of the initial energy of the arrow is lost to air resistance, what was the initial velocity of the arrow?

1) Write an expression for the final energy of the arrow.

$$E_f = U_f + K_f = mgh_f + \left(\frac{1}{2}\right)mv_f^2$$

The total energy of the arrow is its potential energy plus kinetic energy. The potential energy of the arrow is *mgh*. For simplicity, make the height at the firing point equal to zero so that the final height is 5 m.

Remember: *Finding a numerical value at this point is not necessary. Writing an expression will save you time because some terms (usually the mass) may cancel out in a later step.*

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2) Write an expression for the initial energy of the arrow.

$$E_i = U_i + K_i = mgh_i + \left(\frac{1}{2}\right)mv_i^2 = \left(\frac{1}{2}\right)mv_i^2$$

As stated in step 1, the initial height is zero, so the potential energy is zero.

3) Relate the change in energy to the energy lost to air resistance.

$$E_i - E_f = E_{lost}$$

$$\frac{1}{2}mv_i^2 - \left(mgh_f + \left(\frac{1}{2}\right)mv_f^2\right) = E_{lost}$$

The conservation of energy equation tells us that all of the energy of a system must be accounted for. Whatever energy is lost between the beginning and the end must have been due to air resistance.

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4) Relate the energy lost to the initial energy.

$$E_{lost} = (0.1)E_i = 0.1\left(\frac{1}{2}mv_i^2\right)$$

The question states that 10% of the initial energy is lost. Thus, the energy lost is the initial energy times 0.1.

Remember: *In problems with no air resistance (or friction), you can simply set the initial and final energies equal to each other.*

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5) Solve for the initial velocity.
Plug the expression for energy lost from step 4 into the expression in step 3 and solve for velocity.

$$\frac{1}{2}mv_i^2 - \left(mgh_f + \left(\frac{1}{2}\right)mv_f^2\right) = E_{lost}$$

$$\frac{1}{2}mv_i^2 - \left(mgh_f + \left(\frac{1}{2}\right)mv_f^2\right) = 0.1\left(\frac{1}{2}mv_i^2\right)$$

$$\frac{1}{2}v_i^2 - \left(gh_f + \left(\frac{1}{2}\right)v_f^2\right) = 0.1\left(\frac{1}{2}mv_i^2\right)$$

$$\left(gh_f + \left(\frac{1}{2}\right)v_f^2\right) = \frac{1}{2}v_i^2 - 0.1\left(\frac{1}{2}v_i^2\right) = 0.45v_i^2$$

$$v_i^2 = \frac{\left(gh_f + \frac{1}{2}v_f^2\right)}{0.45} = 553.3$$

$$v_i = 23.5 \text{ m/s}$$

Similar Questions

1) A rock is dropped from the top of a 100 m tall building and lands while traveling at a speed of 30 m/s. How much energy was lost due to air resistance?
2) Two different objects are dropped from rest off of a 50 m tall cliff. One lands going 30% faster than the other. The two objects have the same mass. How much more kinetic energy does one object have at the landing than the other?
3) A projectile is fired vertically at a speed of 30.0 m/s. It reaches a maximum height of 44.1 m. What fraction of its initial energy has been lost to air resistance at this point?

Takeaways

Note that the angle and the mass were never used in the calculation. This will often be the case when using the conservation of energy equation to solve problems.

These problems can be worded in many different ways, but the problem-solving process is the same for all of them:

1) Write an expression for the initial energy and an expression for the final energy.
2) If there is friction or air resistance, the difference between final and initial energy is the energy lost to these forces.
3) If there is no friction or air resistance, the energy lost is zero, so set the two expressions equal to each other.
4) Solve for the quantity of interest.

Keep a realistic view of these problems to check your answers for math errors. Note that the initial velocity is faster than the final velocity, which we expect because the target is higher than the firing point and some energy is lost to air resistance.