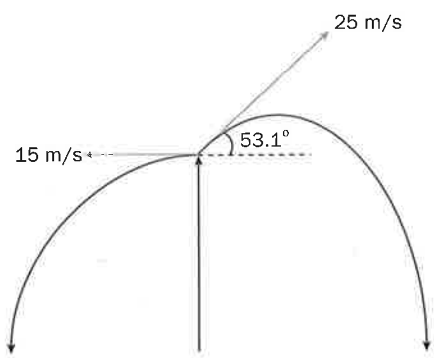
Kinematic Motion

A firecracker with a mass of 100 grams is propelled vertically with a launch force of 1.2 newtons applied over 5 seconds, after which the firecreacker explodes and launches two 50 g fragments. The first fragment goes horizontally to the left with an initial velocity of 15 m/s. The second fragment goes to the right at a 53.1° angle from the horizontal with an initial velocity of 25 m/s. Find the distance between the two fragments once they both hit the ground. Ignore air resistance. ($g = 10$ m/s^2)

1) Draw a free-body diagram of the object's trajectory.



Ignoring air resistance, there are two forces acting on the firecracker initially: gravity and the propulsion provided by the launch. After the explosion, the only force acting is gravity. In our sketch, we must recognize that there are two portions: the linear firing and then the parabolic trajectories of the fragments. The difference between the left and right portions is where the parabola starts.

2) Find the acceleration of the object at launch.

$F = ma$

$F_{net} = ma - mg = m(a - g) = ma_{net}$

$1.2 \text{ N} = (0.1 \text{ kg})a \rightarrow a = 12 \text{ m/s}^2$

$F_{net} = ma - mg = (0.1)(12) - (0.1)(10) = (0.1)(12 - 10) = 0.2 \text{ N}$

Because of Newton's second law, we know that force is the product of mass and acceleration. We can determine the net force either by subtracting the weight (*mg*) from the initial launch force or by finding the acceleration due to the launch force and subtracting out the acceleration due to gravity. Overall, we end up with a 0.2 N force directed upwards and a net acceleration of 2 m/s^2, which is also directed upwards.

Key Concepts

Chapter 1

Newton's second law: $F = ma$ (N: kg · m/s^2)

Kinematics

$v_f = v_0 + at$ (m/s)

$\Delta x = v_0 t + \left(\frac{1}{2}\right)at^2$ (m)

$\Delta x = \frac{v_f + v_0}{2}t$ (m)

Takeaways

In projectile motion problems, you will need to consider the **X** and **Y** components of motion separately. The equations for kinematics are often tested and should be memorized. Another extremely useful equation that was not used here is $v_f^2 = v_0^2 + 2a\Delta x$.

Things to Watch Out For

Don't forget to add all the different forces acting on the object—in this case, the launch force and the weight. The object's mass will not matter for projectile motion when gravity is the only force acting on the object.

3) Find the velocity and distance of the object after the elapsed period.
With the acceleration, we can find the velocity after five seconds. With that velocity, we can find the distance traveled in the first segment of travel. This is simple "plug-and-chug" mathematics.

$$v_f = v_o + at$$
$$\Delta x = v_o t + \left(\frac{1}{2}\right)at^2$$
$$v_f = 0 + 2(5) = 10 \text{ m/s}$$
$$\Delta x = 0(5) + \left(\frac{1}{2}\right)(2)(5)^2 = 25 \text{ m}$$

OR

$$\Delta x = \bar{v}t = \frac{(v_f + v_i)}{2}t$$
$$\Delta x = \left(\frac{1}{2}\right)(10 + 0)(5) = 25 \text{ m}$$

Remember: *The second equation for distance features $\bar{v}$. This is shorthand for average velocity. It would be incorrect simply to use 10 m/s here because the firecracker did not start out with that speed.*

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4) Find the time required for the left fragment to hit the ground.

$$\Delta x = v_o t + \left(\frac{1}{2}\right)at^2$$

Y component: $-25 = 0(t) + \left(\frac{1}{2}\right)(-10)t^2 \rightarrow t = \sqrt{5} \approx 2.2 \text{ s}$

The left fragment has a velocity in the left direction. This will determine how far away it lands but has no bearing on how long it will take the fragment to fall to the ground. The fragment moves in a parabolic curve downward starting from a height of 25 m. Because our final height is 0, this means that the distance we travel is negative. In other words, because we defined gravity as negative, when we move with gravity (downward), we are going in a negative direction. Consider only the **Y** component of motion—no initial velocity and acceleration due to gravity.

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5) Find how far away the left fragment lands.

$$\Delta x = v_o t + \left(\frac{1}{2}\right)at^2$$

X component: $\Delta x = 15(2.2) + \left(\frac{1}{2}\right)(0)(2.2)^2 = 33 \text{ m}$

There is no acceleration due to gravity in the *x*-direction. Because we already found the time that the fragment was falling and we are given its initial velocity, we once again just plug in numbers to the appropriate kinematics formula.

Similar Questions

1) A bottle is thrown with an initial velocity of 4 m/s at 45° from the horizon. Find its final horizontal and vertical velocities before striking the ocean.
2) A pre-1982 penny with a mass of 3.1 grams is dropped from the top of a skyscraper of 1,250 meters. Find the object's acceleration after 5 seconds. If a post-1982 penny with a mass of 2.5 grams is dropped at this point, how far does it travel before the first penny hits the ground?
3) A baseball is thrown vertically with a speed of 3 m/s. Find the total time that the ball has been in flight when it has traveled 50 cm.

Remember: *The horizontal velocity remains constant, so if we instead use* $\Delta x = \bar{v}t = \frac{(v_f + v_i)}{2}t$, *we get the same result.*

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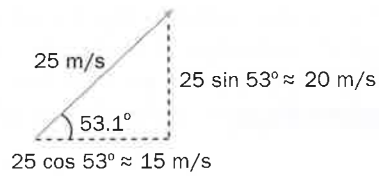
6) Find the X and Y components of the right trajectory.



If the right fragment is shot at 25 m/s at a 53.1° angle, we will need to break its movement into **X** (right) and **Y** (up) components. The **X** and **Y** components behave independently. At this point, it is also wise for us to round our numbers. Thus, we consider a 53° angle and find that $v_x \approx 15$ m/s and $v_y \approx 20$ m/s.

Remember: *When working with sine and cosine on the MCAT, be sure to estimate. Knowing that the sine of 30 is 0.5 whereas the sine of 60 is 0.866, you can estimate the sine of 53 to be approximately 0.8, a little less than 0.866.*

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7) Find the time it takes for the right fragment to hit the ground.

$\Delta x = v_0 t + \left(\frac{1}{2}\right)at^2$

Y component: $-25 = 20t + \left(\frac{1}{2}\right)(-10)t^2$

$\Rightarrow 5t^2 - 20t - 25 = 0$

$\Rightarrow t^2 - 4t - 5 = 0$

$\Rightarrow (t - 5)(t + 1) = 0$

$\Rightarrow t - 5 = 0$ or $t + 1 = 0$

$\Rightarrow t = 5$ or -1

$\Rightarrow t = 5$ s

If we consider the point of explosion to be at a height of zero, the ground is at −25 m. Though we could find the time it takes for the right fragment to peak and return to "zero" or find the maximum height it reaches in this parabola (using $v_f^2 = v_0^2 + 2a\Delta h$, we find that $\Delta h = 20$ m; that is, maximum height above the ground = 25 + 20 = 45 m), it is faster simply to consider the whole trajectory. If the time of explosion is $t = 0$ s, we see that the fragment won't hit the ground until $t = 5$ s. We disregard the negative solution for t since this would refer to a time *before* rather than after the explosion.

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8) Find how far away the right fragment lands.

$$\Delta x = v_o t + \left(\frac{1}{2}\right)at^2$$

X component: $\Delta x = 15(5) + \left(\frac{1}{2}\right)(0)(5)^2 = 75\text{ m}$

Once again, we're only considering the **X** component. We found the time in flight by working with the **Y** component and now we're simply finding the distance that the object can travel in that time.

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9) Determine the final distance between the fragments.

$33 + 75 = 108\text{ m}$

Add the two segments together. The left fragment lands 33 m away from the launching point, 2.2 s after the explosion. The right fragment lands 2.8 s after that (5 s after the explosion), 75 m from the launching point. The total distance between the two fragments is 108 m.
